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TEACHING EUCLIDEAN GEOMETRY USING

PROOF AND DYNAMIC GEOMETRY SOFTWARE

by

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**Chapter 1:** Introduction

## Background

I have taught mathematics at the high school level for the past nine years, in both private and public school districts. During my tenure in secondary education I have taught several courses including geometry. After teaching geometry for five years, I took a graduate level course titled Classical and Modern Geometry; this course opened my eyes to a different way to present the content I had been teaching, utilizing proof and dynamic geometry software. I then found a passion for determining how to instruct geometry to better prepare students to build deeper understanding in order to find success in geometry and future mathematics.

Euclidean geometry is the study of plane and solid figures based on a small number of axioms, the relations between these axioms, and the propositions that logically follow these axioms. The famous work by the Greek mathematician Euclid involved the logical deduction of these propositions (Forder, 1958). Ideas of geometry are useful for solving problems in other areas of mathematics and real-world situations, and has been regarded as a place in the school mathematics curriculum where students learn to reason and see the axiomatic structure of mathematics (NCTM, 2000). Secondary level geometry is instructed in a variety of ways, some involving proofs incorporated throughout, some with proofs as a single unit, and yet others without mention of proof at all (Bossé & Adu-Gyamfi, 2011). Some instructional methods incorporate dynamic geometry software for exploration purposes, some as a method to prove throughout, and others do not incorporate any use. As a geometry instructor, I am looking to determine if the methods I have been using are the best methods, and whether incorporating proof and dynamic geometry software would provide opportunities for better understanding for my students (Editorial Panel, 2011). Using my findings I plan to improve my instruction of geometry and geometric topics incorporating technology, constructions, and proof to align with my research and optimize student learning.

## Statement of the Problem

A few methods of instructing secondary level geometry include:

* building connections in measurement, by beginning with the basic elements in the geometric space through exploration using hands-on materials or dynamic geometry softwares, and ending with formal proof;
* teaching the various theorems directly for application purposes only, not instructing on proofs at all; or
* utilizing some combination of these.

Through discussion with colleagues in my department, we have determined that we do not know which is the best method for facilitating student learning. The problem my research will address is to determine the best method for facilitating student learning which produces deeper understanding for the students and if implementing dynamic geometry software shows to benefit student understanding and application of geometry (Wu, 1996).

## Research Questions

The research questions for this study are as follows:

1. Should secondary geometry instructors include proof as a focus in the instruction of geometry to secondary level students?
2. Does dynamic geometry software implementation impact student performance on geometric concepts and relationships?
3. Does dynamic geometry software implementation impact students’ deductive reasoning and proof?

## Statement of Research Hypothesis / Hypotheses

I hypothesizeby teaching Euclidean geometry to secondary students utilizing proof and implementing dynamic geometry software students will demonstrate deeper understanding of geometric theorems and how to apply them. Using software allows for larger-scale constructions, movement of diagrams to see how properties are related, and the ability to construct without measurement error, which could be involved with paper-pencil constructions. The reference log created by software for each construction allows a path for justification for the end result and will aid students in the development of ideas prior to writing proofs.

## Significance of the Research Problem and Study

Secondary educators of mathematics will benefit by identifying and implementing the best methods for teaching Euclidean geometry. Research involving the use of proof, dynamic geometry software, basic constructions, and other methods has the potential to help secondary teachers provide learning opportunities for students to better understand and build relationships found in geometry. There are many students and teachers who find visualizing the concepts and connecting the concrete to the abstract very difficult. If my research reveals there is a basis for incorporating paper-pencil constructions and/or dynamic geometry software into the classroom in a way that it will benefit the understanding of the concepts and provide a better connection and flow within the course, then I will work to incorporate these elements into my future teaching and share my findings with my colleagues, as well as at mathematics and teaching conferences.

## Limitations and Assumptions

I have limited my research to middle and secondary students and the topic of Euclidean geometry. I am including the middle grades, as the instructional sequence of material differs greatly from one school to another, and geometry concepts are often not isolated to the secondary level. General research regarding the history of geometry and proof will not be restricted to a specific timeline, while research containing qualitative or quantitative studies, will be restricted from 1980, when the first dynamic geometry software was available, to present 2022. There will not be a limitation on location of study as mathematics is studied around the world. I am researching teaching methods and the geometry content with examples from when Euclid began the study to the current teaching practices. Although Euclid was not the only mathematician to contribute to geometry, the most famous work was written by Euclid with an aim to use logical deduction of the propositions of geometry, however, his work has been criticized since it was written (Forder, 1958). I am a teacher in the Midwestern United States, and therefore I have a Western viewpoint of geometry instruction and content.

There are many dynamic geometry software options and I will be researching the use of Cabri Geometry, Geometer’s Sketchpad, and GeoGebra. I am assuming geometry taught in all secondary schools is based on standards implemented from the state and federal government and that there exist standardized tests students are required to take involving the geometry material taught in their classes.

## Definition of Terms

*Deductive Proof:* consist of chains of logical implications connecting the hypothesis to the statement of the conjecture, and characterized by the decontextualization of the ideas presented (Fiallo & Gutierrez, 2017).

*Dynamic geometry environments (DGEs):* particular technology tools that have been used in the learning and teaching of geometry to assist students in moving beyond the specifics of a single drawing to generalizations across figures (Adelabu, Makgato, & Ramaligela, 2019).

*Dynamic geometry software:* software programs, which allow students to manipulate geometric space in an interactive technology setting. The following are commonly utilized: Geometer’s Sketchpad (<https://sketchpad.keycurriculum.com/>), GeoGebra (<https://www.geogebra.org/>), and Cabri Geometry ([www.cabri.com](http://www.cabri.com)) (Andreasen & Haciomeroglu, 2014).

*Empirical Proof:* characterized by showing that the conjecture is true only in one or a few examples taken from a larger set of examples and assuming that the conjecture is also true in all other examples in the set (Fiallo & Gutierrez, 2017).

*Flowchart proof:* A flowchart proof uses boxes and arrows to show the flow of a logical argument. Each reason is below the statement it justifies (Larson & Boswell, 2015).

*Paragraph proof:* A paragraph proof presents the statements and reasons of a proof as sentences in a paragraph. It uses words to explain the logical flow of the argument (Larson & Boswell, 2015).

*Routine proof:* A proof is a logical argument that uses deductive reasoning to show that a statement is true (Larson & Boswell, 2015).

*Two-column proof:* A two-column proof has numbered statements and corresponding reasons that show an argument in a logical order (Larson & Boswell, 2015).

## Summary Statement

Teaching methods of geometry vary from classroom to classroom, and choosing the best instructional methods to promote student understanding of the connections and relationships of measurements in the two-dimensional space is imperative to the success of developing student understanding. In this paper, I explored research on the use of dynamic geometric software and constructing mathematical proofs in geometry in connection with higher levels of student achievement and understanding.

**Chapter 2:** Review of the Literature

## Introduction

“When I was a high school student long ago, the need to study Euclidean geometry was taken for granted” (Wu, 1996, p. 221). Many people could make this statement, as they look back at their time in a secondary school studying geometry. Researchers have shown there are many different ways to approach the instruction of geometry at the secondary level, including proof in multiple forms, constructions using compass and straight edge, and the utilization of dynamic geometric software (Bossé & Adu-Gyamfi, 2011).

Dynamic geometry software environments can provide the means for teachers to create opportunities for students to experiment, make observations regarding the permanence or lack of permanence of many mathematical properties, state, and then verify conjectures more easily than the alternative methods. Students can utilize the software to construct complex figures and perform real-time transformations. This allows students access to a variety of examples that cannot be matched by more traditional environments (Marrades & Gutiérrez, 2000).

## Secondary geometry

There is a range of foci in today’s geometry curricula being taught in our secondary schools; some texts incorporate proof after introduction of the undefined terms and axioms, others use an experimental approach that leaves proofs until the very end of the course, and yet others have no proofs at all (Wu, 1996). Proving geometrical propositions became the norm in the United States for high school students in the 1890s (Herbst, 2002). Proofs have been a part of geometry for centuries, but in curriculum reform have become a lesser focus in more recent curricula, as the focus shifts toward experimentation. Texts that put most or all the weight on experimental geometry are leading students to count on examples to prove truth. Experimentation has an important role in mathematics, but it does not replace the need for formal proof, which supports the idea that mathematics is concerned with statements that are always true. This cannot be certain through only experimentation (Wu, 1996).

Constructions are often incorporated into a single unit of a geometry course rather than utilized throughout for all measurement topics; however, constructions have been shown to help students visualize and understand geometry. Sanders (1998) stated, “geometric constructions can enrich students’ visualization and comprehension of geometry, lay a foundation for analysis and deductive proof, provide opportunities for teachers to address multiple intelligences, and allow students to apply their creativity to mathematics” (p. 554).

Students in secondary geometry may encounter routine proofs and exercises; however, they have the potential to experience a much deeper and more inspiring encounter. Teachers can incorporate a variety of more interactive, nontraditional approaches to proof and discovery of interrelated geometric properties including physical manipulations and use of technology (Posamentier, Colligan, Kalman, & Stallings, 2009).

## Proof in geometry

The National Council of Teachers of Mathematics (NCTM, 2000) directs for all mathematics classes k-12, that students should recognize reasoning and proof as fundamental aspects of mathematics, make and investigate mathematical conjectures, develop and evaluate mathematical arguments and proofs, and select and use various types of reasoning methods of proof. This means reasoning and proof are not special activities, but rather is a natural and ongoing part of classroom discussions, and teachers in classroom environments should convey the importance of knowing the reasons for mathematical patterns and truths (NCTM, 2000).

Two-column proofs, paragraph proofs, and flowchart proofs are common types of proofs used to logically support, or refute, a proposition’s validity. Looking at the history of geometry, the purpose of studying geometry was to grasp the necessary character of the relationships between geometrical objects (Herbst, 2002). For each educator, proofs could hold a different value within the course as a whole and ultimately within the study of mathematics. In the early nineteenth century the expectation involving proofs was that they were part of the mathematical values of the discipline (Weiss, Herbst, & Chen, 2008). Mathematicians, researchers, and educators have questioned over the years whether replicating proofs of the original axioms from Euclid really demonstrates the ability to justify and create an argument. The proof process traditionally taught in United States classrooms is the two-column proof, but there are a variety of options available for teachers to instruct the process of proof outside of this traditional method, by incorporating hands-on techniques and dynamic geometry software. These techniques provide the opportunity for initial exploration, leading to forming a conjecture, and result in the formal final proof of the theorem (Armstrong & McQuillan, 2020).

Teacher attitudes can play a role in whether they are willing to implement alternative strategies and instructional methods when teaching the development of proof, including implementing technology as a tool. Dimmel and Herbst (2017) conducted a study on a voluntary sample of educators involving attitudes toward alternative ways of communication of proofs by students and managing instruction of geometric proofs. The attitudes were analyzed through multimedia surveys investigating secondary mathematics teachers’ reactions to storyboards that represented routine and alternative communication practices of geometry instruction. A group of 73 secondary mathematics teachers, 36 women and 37 men from districts within a 60-mile radius of a large Midwestern university, participated in a virtual breaching experiment study during the 2013-2014 academic year. Participants were shown storyboards of proofs from two categories. Treatment storyboards, which included departures from the hypothesized norms for how teachers expect students to communicate when doing proofs in geometry, and control storyboards, which were designed to represent instruction hypothesized to be routine. Positive or negative reactions to the methods used in the instruction of proofs were recorded. Researchers hypothesized that participants would react negatively to the treatment storyboards, which represented teachers breaching the hypothesized norms, while responses to the control storyboards would have equal positive and negative responses. The results showed that for the treatment storyboards, more responses contained negative statements of attitude than positive statements, where the control storyboards produced nearly equal numbers of positive and negative responses. The conclusion of the study indicated that teacher flexibility toward the management of student presentations of proofs in geometry and the expectation of the communication methods utilized could benefit by allowing for alternatives that would help the students develop and produce adequate understanding and demonstration of mathematical proofs (Dimmel & Herbst, 2017).

Secondary students often struggle with creating a valid proof of a mathematical concept, according to Cirillo and Hummer (2019); this is often a result of many misconceptions or conceptual obstacles. In their study, Cirillo and Hummer (2019) identified five misconceptions, after more than 150 hours of classroom observations of proof instruction in geometry, more than forty interviews with twenty-nine students, posttest results from an eleven-item assessment on proof in geometry with a sample of 389 individuals, and data from a two-week Summer Geometry Institute with eleven students scheduled to study geometric proofs. These five misconceptions are:

1. you can draw conclusions from diagrams,
2. you cannot make assumptions about diagrams,
3. a definition can include all the properties that one knows about the geometric object,
4. bisectors divide triangles in half or act as lines of symmetry, and
5. when attempting to prove a conjecture as a theorem, one assumes the conclusion of the statement.

They were all addressed and when strategies were implemented, classroom teachers found greater success with teaching proof and found less resistance from students as they had a more positive disposition toward learning proof (Cirillo & Hummer, 2019).

Senk (1985) presented data from the Cognitive Development and Achievement in Secondary School Geometry (CDASSG) Project. This project addressed many issues regarding geometry achievement as related to the van Hiele model and was the first large-scale study to determine the level at which secondary geometry students are successful at writing proofs. The study involved a sample of 2699 students from just fewer than 100 geometry classes in thirteen public high schools in five states. This large sample was selected to represent a cross section nationally of educational and socioeconomic conditions. A subsample of 1520 ninth-eleventh grade students from eleven schools in five states was selected from schools and classes that had studied proof writing, and with teachers willing to conduct the testing. Student achievement was assessed using three nonoverlapping forms of a six-item test on proof. These tests were developed by the researcher and informed by comments from classroom teachers and the CDASSG staff utilizing pilot studies to ensure content validity, clarity of directions, and adequacy of time allotted. Regular classroom teachers administered the achievement tests about one month before the end of the school year, and each proof was graded blindly by two teachers on a scale from zero to four. Each test was given a total score as a sum of the item scores, with a maximum of 24 points, and the number of correct proofs, with a maximum score of four per item. Results showed that a few students did exceptionally well at writing proofs, as three percent of students received perfect scores and approximately 30 percent scored at a 75 percent mastery level of writing proofs. Overall results showed 25 percent have virtually no competence, 25 percent can only complete trivial proofs, about 20 percent can do some proofs of greater complexity, while, 30 percent master proofs in standard textbooks. This demonstrates a rather low level of achievement in proof writing (Senk, 1985).

Pandiscio (2002), conducted a qualitative study on how four preservice mathematics teachers perceive the need for and benefits of formal proof, when given geometric tasks in the context of dynamic geometry software. At the beginning of the study, all four responded that proof is an important part of geometry in the initial questionnaire; however, after completing the tasks presented to them using Geometer’s Sketchpad, participants were unsure students would find any value in preparing a formal proof due to the software’s ability to produce numerous varied examples for a given situation (Pandiscio, 2002).

## Dynamic Geometry Software

“Using dynamic geometry software, students can quickly generate and explore a range of geometric examples” (NCTM, 2000, p. 311). Three main types of computer environments are static construction environments, Logo-based environments, and interactive or dynamic geometry environments (DGEs). Prior to DGEs, static construction environments such as Geometric Supposer typically allowed students to construct geometric objects to then apply common Euclidean constructions. It would record a sequence of constructions and then apply the sequence to a new object, but would not allow students to manipulate the original object directly to observe the effects of the manipulations in real time. Also prior to DGEs, Logo-based environments allowed students to create geometric objects and then apply constructions and transformations to the objects using a series of pseudo-programming commands, but again did not tend to automatically update the results of constructions or transformations when students changed the underlying geometric object. Later, interactive or dynamic geometry environments allowed students to construct an original geometric object, alter that object with the results of the alteration, then apply to all constructions and transformations, and immediately update on the screen (Glass & Deckert, 2001). With DGEs, there is a potential that students would not be required to demonstrate understanding prior to constructing, but it can be utilized to ensure students have deeper understanding in order that the construction, when manipulated, stays true to the characteristics required for the figure. With Geometric Supposer software, students have the opportunity to create static figures, and then implement constructions on them to test conjectures to be implemented and tested on a variety of figures. Each software has the potential to produce deeper understanding of the geometric relationships (Battista, 2007). Giamati (1995) stated “explorations that give students the opportunity to make reasonable conjectures deepen the students’ understanding of what it is to *do* mathematics” (p. 458).

Dynamic geometry environments provide students the ability to create conjectures, test them, and then create an algebraic form. This allows students to present a partially geometric proof where the algebraic manipulation was done for them, removing the stumbling block the algebraic process can create (Lyublinskaya & Funsch, 2012). However, dynamic geometry software first must be introduced to the students with an opportunity to use the software before beginning a task where students will be asked to prove a conjecture in geometry. The GeoGebra software tends to have a quick learning curve for students and is fairly intuitive (Bayaga, Mthethwa, Bossé, & Williams, 2019). Geometer’s Sketchpad allows students the opportunity to explore theorems ranging in complexity, but the software takes time to learn (Giamati, 1995).

Dynamic geometry environments can be appropriate tools when implemented in middle or secondary school geometry courses. These tools have the ability to aid students to make sense of the structure of mathematics (Andreasen & Haciomeroglu, 2014). Although paper-pencil constructions using a compass and a straightedge are still valuable, dynamic geometry software reduces the amount of time required for these constructions. The software also allows students the opportunity to manipulate and explore more complex problems than a teacher-led demonstration or a physical construction would allow. Through Andreasen and Haciomerglu’s (2014) experience with mathematics education in the middle and high school settings, they discussed the many capabilities of the software. Using software, such as GeoGebra or Geometer’s Sketchpad, students were able to not only explore geometric properties and relationships but were also able to extend the understanding into applications. Prior to the dynamic geometry environments, students would examine relationships within geometric figures by watching teacher demonstrations, completing time-consuming constructions by hand, or simply applying theorems rather than discovering theorems. These instructors found the software was useful for instruction, understanding, and assessment of student learning as these dynamic geometry software options help students make sense of the structure of mathematics (Andreasen & Haciomeroglu, 2014).

## Student Achievement

A study conducted in South Africa by Bayaga et al. (2019), considered student achievement on pretest and posttest data of Euclidean geometry material between two groups of students. The control group, instructed traditionally with direct instruction and the experimental group, instructed utilizing GeoGebra dynamic geometry software. The sample of participants were 11th grade students chosen from three mathematics classes in a school recognized as high poverty, and known for its lack of access to technology. There was a total of 112 students, 56 assigned to the control group and 56 assigned to the experimental group. The independent variable for this study was the instructional methods the groups received: traditional instruction, described as direct, lecture-based, and on a static chalkboard for the control group, while the experimental group received instruction incorporating GeoGebra, including a 90-minute session introducing students to the techniques of constructions and measurement within the GeoGebra platform. The dependent variables for the study were achievement on the pretest compared to the posttest, and the attitudes of the participants according to the results of a questionnaire used to find student’s views and attitude toward the use of GeoGebra. The procedures included a pretest administered to both groups, followed by instructional methods delivered for a one-week unit, at which time the students were administered the posttest as well as a questionnaire collecting data on the students’ evaluations of the software, and how its use effected their growth of understanding. The pretest/posttest, consisting of ten questions, was developed targeting the processes of problem solving, critical interpretation, and evaluative judgments. The quality of the data produced was ensured as the researchers first field-tested the content and instruments with populations similar to the chosen sample. An independent sample t-test was used on student scores for the pretest and posttest between the control and experimental groups, to assess the impact of GeoGebra compared with the traditional instruction methods. The analysis of the pretest data found a mean score of 5.5/10 for the control group and 6.36/10 for the experimental group. The p-value was 0.1083221, which is greater than the chosen significance level of 0.05. Therefore, there was no statistically significant difference between these mean scores. The analysis of the posttest produced a mean score of 7.52/10 for the control group and 9.45/10 for the experimental group with a p-value of 0.000027, which is less than the chosen significance level of 0.05. This demonstrated that the difference in these means is statistically significant. A paired samples t-test to compare the pretest and posttest scores, resulted in a 3.09 mean score difference of the pretest and posttest for the experimental group, and a 2.02 for the control group. The t-value was 13.7 and the p-value was 0.000, which is less than the chosen significance level of 0.05.

Results of the analysis of the pretest mean scores between the control group and the experimental group indicated the two groups were comparable at the beginning of the study, while the posttest analysis demonstrated the experimental group mean score was statistically different when compared to the control group mean score. Analysis of the pretest data to posttest data, there was a statistically significant difference, this indicated that differences between the two groups scores is likely due to the treatment of using GeoGebra (Bayaga et al., 2019).

Hollebrands (2007) conducted a qualitative study, which analyzed the strategies students utilized when working with dynamic geometry software in a seven-week unit on geometric transformations. The study involved 16 students in a tenth-grade honors geometry course, who attended a public high school, with an enrollment of approximately 1000 students. The high school was located near a large university. The students varied socioeconomically, but exhibited little racial diversity, as the majority of the students were Caucasian and spoke English as their first language. Of the 16 students, six were selected as participants to reflect a range of mathematical abilities, and case studies were conducted focusing on four of the six for consistency as they had the fewest absences from class. Each student had consistent access to Geometer’s Sketchpad (GSP) on computers in their classrooms, and was familiar with the software use. The study incorporated film of the students and discussions regarding their thinking process throughout an instructional time. Students in the study had extensive experience writing conjectures, engaging in guided explorations and constructing explanations through their previous study on topics of similarity, congruence, triangles, and quadrilaterals. Participants were interviewed first to identify prior understandings of transformations; technology was not used in this interview. Second and third interviews were conducted where students had access to GSP. While being interviewed, students worked on tasks related to geometric transformations, some involved premade GSP sketches, while others required students to construct their own sketches from scratch. The two strategies evident in student processing were what the researcher described as reactive and proactive strategies. These strategies were in relationship to the student’s ability to anticipate the result of their action using the technology, to reflect and interpret the result, and then perform a further action. Students’ strategy use depended on their understanding of the technology tool as well as the mathematical concepts being incorporated. The reactive strategy indicated a student may not know what their action will produce, thus their choice of action is dependent upon what their previous action produced. The proactive strategy is usually guided by their understandings of the geometric properties and relations as they anticipate the result of their action and have utilized a plan upon which they later reflect. This research study found student understanding was demonstrated by their strategy and use of the software. The use of technology seemed to support the development of mathematical understandings (Hollebrands, 2007).

There are multiple ways to incorporate technology including dynamic geometry software into a classroom, one way is through instructor directed steps or completely controlled by the instructor. Students simply watch the manipulation of the figures completed by the instructor. A study by Bokosmaty, Mavilidi, and Paas (2017) examined the effects of using Dynamic Geometry Software Cabri to manipulate shapes to determine geometric properties, or the observation of manipulations on learning geometric properties. The study involved 60 students in year five (ages ten and eleven) at a private school in an Australian city. Students were placed into three groups and instructed in two 45-minutes sessions on the content and two 45-minute sessions for software training. One group was directed by their instructor to manipulate specific geometric measures, another group watched the teacher use the software to manipulate the geometric measures, and the last group was instructed using static pictures demonstrating the geometric measures. The analysis of this study was completed using the instructional method as the independent variable. The dependent variables were the similar test scores (a test with problems similar to the instructional examples), transfer test scores (a test with problems that required the students to transfer their knowledge to a different orientation of the geometric figure), and subjective ratings of the cognitive load for the subjects. The analysis showed the transfer test results were similar to the results for the similar test. Through statistical analysis, results indicated a significant difference between the manipulation group and the conventional group. The manipulation group outperformed the conventional group, as well as the observing manipulation group outperformed the conventional group. However, there was no significant difference between the manipulation group and the observing manipulation group for the transfer test. This study concluded the manipulations of geometric properties by students through mouse movements, or observing these manipulations would enhance learning of geometric properties compared to those instructed using static pictures (Bokosmaty, Mavilidi, & Paas, 2017).

Another study, conducted by Adelabu, Makgato, and Ramaligela (2019), examined the importance of dynamic geometry software on a convenience sample of 87 ninth grade students in two schools in South Africa over eight weeks. This study assigned the control group to be from one school and the experimental group from the other, due to the experimental school having access to computers for the dynamic geometry software. The groups did not present statistically significantly different mean scores on the pretest but did present statistically significant posttest scores after a treatment for the experimental group of instruction incorporating GeoGebra, a dynamic geometry software. The control group was instructed using a textbook and a chalkboard for instructional activities. The t-test result for the pretest was a p = 0.433 which does not satisfy the statistical significance of p < 0.05, and indicates the samples were similar before the experiment; however, the t-test results for the posttest was p = 0.0004, which indicates the statistically significant difference between the two scores. The analysis by researchers determined the implementation of GeoGebra for the experimental group demonstrated improved achievement (Adelabu, Makgato, & Ramaligela, 2019).

A study to determine if computer-augmented instruction increased the knowledge gains for geometry students by Funkhouser (2002) involved an early Dynamic Geometry Software called The Geometric Supposer and 49 participants from geometry courses at a large public high school in the western United States. This lengthy study took place over two 18-week semesters with a control group of 27 students instructed in plane geometry using a traditional, noncomputer-based approach, and a treatment group of 22 students instructed in plane geometry incorporating activities with the Geometric Supposer software. These groups consisted of students with a wide range of abilities with passing grades in Algebra 1. These groups were found comparable through the analysis of the mean final grades for the students in their Algebra 1 course. The mean final grades for the control group was 3.17 out of 4 and 3.27 out of 4 for the treatment group; the t-test results for the mean scores produced a difference that were not statistically significant at the 0.05 level of significance. Both groups were assessed through a standardized test of geometry to measure understanding after the instructional units were completed. Each student was assigned a geometry performance score totaling the number of correct responses on the standardized test. Analysis was conducted using the means and standard deviations for both groups and differences were analyzed using a t-test. The mean scores were 34.26 out of 50 for the control group and 37.00 out of 50 for the treatment group, and the t-test results showed a significantly better performance from the treatment group than the control group based on 0.05 level of statistical significance. The results of this study showed students who received geometry instruction implementing computer-augmented strategies and activities demonstrated greater gains in knowledge of geometry concepts than students who received traditional instruction (Funkhouser, 2002).

Erbas and Yenmez (2011) conducted a study in a suburban district of Ankara, Turkey involving 134 sixth-grade students in a public school with the majority of students being low-to middle-class families. The purpose of this study was to determine the effects of using a DGE, Geometer’s Sketchpad, in an inquiry-based learning environment on student’s understanding of the congruency and similarity of polygons compared to the traditional, direct instruction methods often used. One of the researchers was the instructor of the geometry courses with 21 years of teaching experience, and the other participated as an observer, offered technical assistance if a student needed help with Geometer’s Sketchpad, and asked students questions about their thinking throughout the instructional periods. This two-week study had a control group of 68 students, and an experimental group of 66 students. Each group completed the 20-question achievement test as a pretest. The test was developed to measure the achievement in, and understanding of, similarity and congruency of polygons, and incorporated multiple choice, matching, true-false, open-ended, and short answer items as a pretest. Both groups participated at the same time and met twice a week in a block session 80 minutes long. The experimental group had instruction prior to the sessions regarding the use of the Geometer’s Sketchpad by the researcher for two class hours totaling 80 minutes of additional instruction. Researchers felt this was adequate as students had previous knowledge of basic computer skills. Students were paired up to work on ten worksheets, designed by the researchers, as adaptations of the inquiry-based activities in the textbook the control group was working with. Both groups were completing activities with the same learning objectives, only the materials were different, the Geometer’s Sketchpad versus hands-on materials including a ruler and a protractor. These activities were designed to guide students through a period of discovery to make observations and generalizations in order to find a conjecture. They would then test the conjecture to draw conclusions regarding the similarity and congruence of polygons. In both the experimental and control groups, after students finished each worksheet and wrote down their findings, there was a class discussion about the activities and conclusions when students could take additional notes and finish any remaining parts of each worksheet. At the conclusion of the two-week period, each group was administered the achievement test as a posttest and a delayed posttest three months after the treatment was discontinued. The results were analyzed, and the two-week long treatment showed substantial improvement in students’ achievement in the experimental group compared to the control group. Researchers also found the use of the DGE, Geometer’s Sketchpad, had a significant effect on students’ retention, measured by the delayed post-test (Erbas & Yenmez, 2011).

## Dynamic Geometry Software and Proof

Few studies have been conducted to determine if dynamic geometry software would lead students to greater understanding of deductive reasoning and proof, or if it would negate the necessity of writing formal proofs in a secondary geometry setting. Jones (2000) studied 12-year-olds over a nine-month period as they investigated the classification of quadrilaterals. This qualitative study focused on students’ sense of the software when examining their interpretations and explanations, to determine if students’ explanations can grow from imprecise to mathematical explanations. This growth, mediated by the software environment, could help provide a foundation to build notions of deductive reasoning involved in formal proofs. Pairs or groups of students took turns working on tasks regarding the relationships found between quadrilaterals where they were asked to construct figures that would maintain certain characteristics when dragged with dynamic geometry software, Cabri Geometry, on four computers available. Through the discussion with the students Jones found that initially students emphasized descriptions rather than explanations, with some reliance on perception rather than using mathematical reasoning. As students progressed, explanations were found to be more mathematically precise, although some student explanations were influenced by language specific to the software and its dynamic nature. By the end students had transitioned to explanations entirely related to the mathematical context. Results of this study showed using dynamic geometry software does provide students access to the world of geometrical theorems through carefully designed tasks, teacher involvement, an encouraging environment in terms of conjecturing, and focus on mathematical explanations. This can lead to developing deductive reasoning, however, if conducted without these aspects, use of the software has the potential to reduce the perceived need for deductive proof (Jones, 2000).

Research studies are emerging regarding the impact of dynamic geometry software use on student understanding and development of deductive proof. Marrades and Gutiérrez (2000) worked with a group of 16 students aged 15-16 years old in a secondary school on a geometry unit based on dynamic geometry software Cabri. The purpose of this 30-week study with two 55-minute sessions per week was to investigate how DGEs can help student conception of proof in mathematics improve as well as their methods of justification. This teaching experiment took place as part of the ordinary mathematics teaching with their own teacher, who was also one of the researchers, and during the standard class time. Students worked in pairs on a set of computers with Cabri-Geometry (version 1.7) and had previous knowledge working with Cabri to solve conjecture problems with this teacher from the previous year, which set the background necessary for this teaching experiment. The follow-up of the study was carried out with two pairs of students, all boys, who represented high to average abilities and attitudes, and who were selected by the teacher prior to beginning the experiment. The teaching unit was comprised of 30 activities, each structured with a beginning phase where students use Cabri to create or open a figure and explore its characteristics, the second phase asked students to generate conjectures or verify conjectures given to them. In certain activities the last phase asked students to justify their conjectures. Activities were presented to students in the form of a worksheet, where they had space to write any observations, comments, and their conjectures and justifications. Each activity was structured to maximize the benefit of using the dynamic capabilities of Cabri and involved the action of dragging parts of figures as a central role for generating and checking conjectures. The results of this qualitative study, with discussion and interaction with the two pairs of students, showed students with the highest academic ability improved the quality of their justification skills and other students made limited progress or no progress, indicating that secondary students require a significant amount of time to begin to feel confident with deductive justifications and formal proofs. Students showed progress in the ability to produce justifications or proofs when parallel learning of mathematical concepts and properties were being studied. This was recognized as students failed to solve a problem because they did not remember or had not learned necessary geometric properties. In this experiment, dragging geometric objects allowed students to see many examples in just a few seconds and provided them with immediate feedback they could not have obtained through paper-pencil constructions. Researchers found dragging guided students to look for properties, special cases, and counterexamples that could be connected to form a conjecture or a justification (Marrades & Gutiérrez, 2000).

Fiallo and Gutiérrez (2017) conducted a qualitative analysis on 17 grade 10 students in a secondary school in Floridablanca (Santander, Columbia) from a trigonometry course generation of deductive proof utilizing Cabri to form conjectures and formalize proofs using both empirical and deductive mathematical argumentations. Empirical proofs were defined as showing a conjecture is true using one or a few examples taken from a larger set of examples, and deductive proofs consist of logical implications connecting a hypothesis to the statement of the conjecture. The study was conducted during the ordinary classroom schedule by their teacher with the other researcher in the classroom as a participant observer who was also available to record observations, answer student questions, and inquire about their work. Students worked in pairs with one group of three. Each group had a computer with Cabri available during the four-month intervention made up of two 90-minute sessions per week in the computer room and a 45-minute session per week in either the computer room or a regular classroom. Students and teacher were familiar with Cabri software and had adequate previous geometry knowledge, which was necessary for the completion of the conjecture and proof problems. Two pairs of female students of average academic level, who asked questions and with high participation were chosen to be video recorded throughout each of the sessions in order to collect information about the students’ productions and activity. Students responded to questions regarding their thought process during both successes and failures while solving conjecture-and-proof problems, and results demonstrated students began producing simple empirical proofs, but ended up producing deductive proofs. Researchers found two relevant factors contributing to this improvement were the dynamic geometry software environment, which helped students to discover and verify conjectures through the ability to manipulate figures quickly and accurately, along with the teaching methodology of discussion and justification for answers. Although dynamic geometry software was not the primary focus of this study, Fiallo and Gutiérrez (2017) found it to play a role in the success of proof writing for the students.

## Summary Statement

Secondary geometry is a course which can be instructed using a variety of methods; mathematical proof is often incorporated into the curriculum, and many researchers have studied the use of dynamic geometry software to facilitate students’ learning geometric relationships. The content in a geometry course varies greatly in the incorporation of proof, constructions, experimentation, direct instruction, and dynamic geometry software.

The literature reviewed included research on secondary geometry content and teaching methods, proof and construction use in geometry, dynamic geometry software, and student understanding of geometric concepts and proofs.

**Chapter 3:** Findings

## Introduction

Through review of literature related to these three research questions:

1. Should secondary geometry instructors include proof as a focus in the instruction of geometry to secondary level students?
2. Does dynamic geometry software implementation impact student performance on geometric concepts and relationships?
3. Does dynamic geometry software implementation impact students’ deductive reasoning and proof?

Studies provided insight into secondary geometry instruction, and here I analyzed the findings regarding writing proofs, utilizing dynamic geometry software to provide students the opportunity to improve performance on geometric concepts and relationships, and the impact of dynamic geometry software use on students’ development of deductive reasoning and proof.

## Results

*Question 1: Should secondary geometry instructors include proof as a focus in the instruction of geometry to secondary level students?*

Do researchers agree that writing proofs has remained an important role in secondary geometry instruction? Euclid wrote his book of the elements as he proved the relationships throughout his study of measurement, but researchers today do not agree that proof is still a central and necessary part of geometry as it is taught in the secondary school systems around the world. NCTM standards say students should recognize reasoning and proof as fundamental aspects of mathematics, and historically proofs have been a foundation as students grasp the necessary character of the relationships between geometrical objects (NCTM, 2000). While the expectation in the early nineteenth century was that proofs were a part of the mathematical values of the disciplines, studies have shown teachers have different ideas of what proof looks like in their classrooms (Herbst, 2000; Weiss, Herbst, & Chen, 2008). Through studies and experience, researchers have found that students in full year geometry courses find little success when writing valid proofs (Senk, 1985). This has been attributed to teacher attitudes and willingness to implement alternative strategies and instructional methods, including technology (Dimmel & Herbst, 2017), and student misconceptions that tend to hinder proof writing success and cause students to struggle (Cirillo & Hummer, 2019).

As geometry’s curricular focus shifts from formal proofs toward experimentation, including the use of dynamic geometry software, researchers do not agree the need for formal proof has remained constant. Preservice teachers who began a study believing proof writing was an important part of geometry, later were unsure if students would continue to find value in preparing formal proofs after completing tasks investigating geometric relationships and concepts utilizing Geometer’s Sketchpad (Pandiscio, 2002). Wu (1996) believed experimentation does not replace the need for formal proof, and Jones (2000) found that when using dynamic geometry software with carefully designed tasks, teacher involvement, an encouraging environment promoting conjecturing and a focus on mathematical explanations, the need for deductive proof was not reduced (Jones, 2000).

*Question 2: Does dynamic geometry software implementation impact student performance on geometric concepts and relationships?*

Have researchers found evidence that incorporating dynamic geometry software in geometry instruction leads students to demonstrate greater understanding of the geometric concepts and relationships? Several studies investigating how dynamic geometry software use impacts student achievement have been conducted in multiple countries (e.g., Adelabu, Makgato, & Ramaligela, 2019; Bayaga, 2019; Bokosmaty, Mavilidi, & Paas, 2017; Erbas & Yenmez, 2011; Funkhouser, 2002). Thus, these studies have involved diverse student populations regarding socioeconomic levels, ages, and mathematical abilities. Studies utilized a variety of dynamic geometry software options including: GeoGebra, Geometer’s Sketchpad, Cabri, and the Geometric Supposer, while duration ranged from a one-week to two 18-week semesters. In general, these quantitative studies found student achievement was higher on a posttest regarding geometric concepts and relationships for an experimental group using dynamic geometry software, compared to the control group instructed using traditional methods. Researchers agree that utilizing dynamic geometry software leads to greater understanding of geometric concepts and relationships as demonstrated through a pretest-posttest analysis (Adelabu, Makgato, & Ramaligela, 2019; Bayaga, 2019; Bokosmaty, Mavilidi, & Paas, 2017; Erbas & Yenmez, 2011; Funkhouser, 2002). A qualitative analysis by Hollebrands (2007) showed that dynamic geometry software supported the development of understanding through their use of reactive and proactive strategies. Their choice of strategy was based on their ability to anticipate the results of their actions while using the technology to reflect and interpret results.

*Question 3: Does dynamic geometry software implementation impact student’s deductive reasoning and proof?*

Have researchers found evidence that dynamic geometry software promotes increased achievement of deductive reasoning and writing valid proofs? Minimal studies have been published directly investigating the use of dynamic geometry software and increased achievement on writing proofs (e.g., Fiallo & Gutierrez, 2017; Marrades & Gutierrez, 2000). However, researchers agree that with sufficient time and adequate use of dynamic geometry software incorporated into geometry instruction, students showed improved justification skills and the generation of deductive reasoning and successful proof writing. Two qualitative studies showed use of Cabri, with its action of dragging figures as activities guided students to generate or check conjectures and then justify conjectures, led higher ability students to improve the quality of their justification skills. Other students with lower mathematical ability demonstrated limited or no progress. Both studies showed improved quality of justification skills and the generation of deductive proof to form conjectures and formalize successful proofs (Fiallo & Gutierrez, 2017; Marrades & Gutierrez, 2000).

## Summary Statement

In this chapter I have completed a synthesis and presented the findings from my analysis of the literature as they related to my research questions. Question 1: Should secondary instructors include proof as a focus in the instruction of geometry to secondary level students? Researchers have generally agreed that proof should remain a focus in the instruction of secondary geometry. Question 2: Does dynamic geometry software implementation impact student performance on geometric concepts and relationships? Yes, quantitative studies concluded that student achievement was impacted using dynamic geometry software when compared to students instructed by traditional textbook methods. Question 3: Does dynamic geometry software implementation impact student’s deductive reasoning and proof? Inconclusive, further research is necessary to determine if dynamic geometry software use impacts student’s deductive reasoning and proof.

I will discuss contradictions, inconclusiveness and my conclusions in the next chapter along with any implications and applications for my own classroom and to share with other secondary math educators.

**Chapter 4:** Discussion and Conclusion

## Introduction

As I critiqued the literature found involving secondary geometry, proof in geometry, dynamic geometry software, and the impact incorporating dynamic geometry software has on student achievement, I determined there were a few issues that concerned me. In this chapter I will discuss these issues including the variety of softwares used, length of studies, selection of participants, ages of participants, geographic location, multiple variables involved, and contradictory and inconclusive results. I will conclude by discussing the implications and applications of these results in my personal classroom teaching.

## Discussion

### **Issues with the Research**

Research studies regarding the use of dynamic geometry software were inconsistent regarding software program used. This is likely because geometry softwares have changed over time. Availability and accessibility to some programs has become difficult due to device compatibility, being discontinued, or cost. Although Cabri Geometry, Geometric Supposer, Geometer’s Sketchpad, GeoGebra, and Logo are all types of geometry software, they are not all created equal in quality, ease of use, cost, or accessibility on common devices available in geometry classrooms. I considered these differences when comparing results from these studies, and although the software differed, the intentions of the studies were comparable.

Other further concerns were the limitation of time for studies and the selection of participants primarily due to availability of computer and software access rather than a carefully selected experimental design. The lengths of many of the studies were quite short and insufficient to constitute adequate learning opportunities for the students to truly demonstrate understanding, and the selection of participants was not broad to include a variety of learners. These specifics raise concern for the generalizability of results, as researchers noted further research would be beneficial with an extended time frame for study and a larger selection of participants.

I did not restrict research studies to the United States, which led to the ages of participants being inconsistent. When considering if the ages of the participants changed the validity of the results and their application, I determined studies involving younger participants were valid as the specific geometry content remained consistent, while course sequencing in different areas was variant. Thus, ages of students would be inconsistent when compared to the midwestern United States where geometry is traditionally instructed to students in their ninth or tenth grade school year.

Results from a few studies were more difficult to evaluate due to circumstances requiring participants to share computers with dynamic geometry software which introduced an additional variable of collaboration. These studies mentioned this as a factor for further study, as it is difficult to accurately conclude which variables truly led to the results of the study. Researchers noted in their studies when this had occurred and the need for further studies to determine the true causation involved.

### **Contradictory Results**

Literature involving secondary geometry and proof in geometry varied greatly and at times was inconclusive or contradictory between researchers on expectations in a geometry course. Tradition, culture, geographic location, and teacher discretion all played roles in these differing expectations for content and methods involving proof, explorations, constructions and use of technology. As culture continues to change, incorporating formal proofs in secondary geometry courses is likely to continue to morph and change also. NCTM standards state students should recognize reasoning and proof as a fundamental aspect of mathematics for all ages, which implies that proof should play a crucial role in all mathematics courses including secondary geometry (NCTM, 2000). Wu (1996) and Jones (2000) separately concluded the need for formal deductive proofs, along with the expectation in the early nineteenth century where proofs were part of the mathematical values of the disciplines (Herbst, 2000; Weiss, Herbst, & Chen, 2008). Similar conclusions have led other researchers to consider why, through studies and experience, students in full year geometry courses find little success when writing valid proofs (Senk, 1985).

Through my analysis, I have determined researchers generally agree formal proof to be a key aspect of demonstrating knowledge of mathematical understanding and reasoning. However, students often find little success when writing valid proofs and may find little value in doing so. Cirillo and Hummer (2019) outlined common misconceptions they found students made that hindered their ability to write accurate proofs and lead to a negative disposition toward learning proofs (Cirillo & Hummer, 2019). This can in turn lead educators to negative attitudes toward the topic, to neglect the topic altogether, or to possibly implement alternative strategies of instruction including technology. I came across contradictory results on the necessity of formal proof when Pandiscio (2002) found preservice teachers who began a study believing proof to be important later were unsure if students would be persuaded that formal proof would not be necessary with the experimentation and many examples dynamic geometry environments provided. This result ultimately did not lead the researcher to believe formal proof played a lesser role in secondary geometry, but rather the use of DGEs could prove to create an environment that is more difficult to lead students to formal proof from investigation. My conclusion is researchers agree formal proof remains an important fundamental aspect of secondary geometry as well as other mathematics.

### **Inconclusive Results and Further Research**

Research studies and findings on the implementation of dynamic geometry environments and their impacts, specifically on student’s deductive reasoning and proof are limited. Two qualitative studies were conducted using Cabri Geometry and findings led researchers to conclude that with adequate use of software higher ability students demonstrated improved quality of justification skills. As minimal research has been done in this specific area of dynamic geometry software use for improved deductive reasoning and proof, and as researchers have concluded the importance of formal proof, further research would be beneficial here. Due to the lack of research currently, I am not able to conclude if the use of dynamic geometry software impacts student’s ability to produce formal deductive proofs in secondary geometry courses.

## Conclusion

My classroom experiences teaching proof and using dynamic geometry software have produced both successes and failures. I have been caught between my own experience learning geometry at the secondary level, the undergraduate level and the graduate level. Each level was taught by different instructors with various experience and methods of instruction. My secondary level course involved minimal paper-pencil constructions, no geometry software use, one section of formal proofs and almost no connections with the study of measurement throughout the course. My undergraduate level course was focused more on paper-pencil constructions, some geometry software use, and many formal proofs. However, it remained to be somewhat-disconnected sections of content combined to create geometry. My graduate level course was founded on constructing using both paper-pencil and dynamic geometry software, formal proofs for all content, and building the course through connections throughout the entire course. This last course opened my eyes to the possibilities within my own classroom.

I began teaching geometry a couple years after completing the undergraduate level course and followed a traditional textbook. This textbook placed very little emphasis on formal proof or constructions, and due to time restrictions and standards, these aspects unfortunately were squeezed in occasionally. Once I completed the graduate level course, I had a much deeper understanding and appreciation for geometry, viewed now as the study of measurement, rather than as a conglomeration of theorems used to complete textbook problems to find missing measures.

Research conclusions regarding secondary geometry, the importance of proof in geometry, and the use of dynamic geometry software has sparked a new vision within me for my classroom. As studies have shown that student achievement is positively impacted by the use of dynamic geometry software and the resulting investigative environment, I am interested in incorporating this into my classroom and building a desire in students to investigate to convince themselves, then to convince their friends, and ultimately to prove in general the geometric relationships they explore.

In order to properly incorporate the use of dynamic geometry environments in my classroom a few things need attention. I will need to find the appropriate software that is accessible for my students and become acquainted with the software and its uses. I will also need to determine beneficial use of the software for both instruction and understanding and how to best incorporate its use in my planning of instructional lessons for the best results. Working with my colleagues, curriculum director, and administration will be helpful as we work toward greater understanding and better use of resources and teaching methods available to us and to our students.

## Summary Statement

After presenting the research findings, discussing and interpreting the results, and concluding with the implications of the results for me, my students, my colleagues and for future research, I am excited to apply this research to my classroom. Education is always changing; methods and tools are evolving and often available. I conclude dynamic geometry software and formal proofs are worth incorporating in my classroom for the betterment of my instruction and the positive impact research shows it will have on the understanding my students will have of geometry, the study of measurement.

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