

Graduate Studies Office
Bemidji State University
1500 Birchmont DR NE
Bemidji, MN 56601
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A COMPARATIVE ANALYSIS OF JAPANESE AND
U.S. TEACHING STYLES OF MATHEMATICS

by

Timothy Michael Wurdock

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THIS RESEARCH PAPER HAS BEEN APPROVED ON THE DATE SHOWN BELOW:

Dr. Randy Westhoff,
Committee Chair
Professor of Mathematics

Date

Dean of Graduate Studies

Date

A COMPARATIVE ANALYSIS OF JAPANESE AND
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Timothy Michael Wurdock

This study examines the effects of different international teaching styles on the performance of junior high school students. Japanese students ranked higher than U.S. students on international tests. The whole-class method and open-ended problems are techniques frequently used in Japanese classrooms. These techniques will be incorporated into the author's curriculum to determine their effectiveness in teaching mathematics. Two eighth grade classes will be studied. The author will teach the experimental group using the whole-class method. The control group will be taught using the American method.

Approved by:

Committee Chair

Date

Committee Member

Committee Member

Graduate Faculty Representative

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CHAPTER I

INTRODUCTION

Description of the Problem

The purpose of this research paper is to study the effects of differing international teaching styles on the performance of junior high school students. This study will evaluate teaching methods commonly used by American instructors and compare them to those used principally by Japanese teachers. The Third International Mathematics and Science Study (TIMSS) will be used to compare different teaching methods in these countries. The techniques used by Japanese instructors will be incorporated into the author's curriculum and the results will be described in Chapter III.

Specifically, this paper will describe the differences in the two systems and describe the effects of changing a typical U.S. lesson to model a Japanese lesson. I will then discuss the practicality of permanently incorporating these Japanese teaching strategies into American curricula. Finally, this paper will examine the effects Japanese teaching styles have on students' performance in the subject of mathematics.

Background of the Problem

TIMSS revealed that American students rank near the middle of the spectrum of international comparisons of mathematical skills. Consequently, the results of this test raise concerns about the mathematical ability of U.S. students. In contrast, Japanese students were the third highest scoring group on the test. TIMSS also highlighted several differences between the educational systems in Japan and the U.S., such as curriculum,

In comparing the differences between the two systems, some key questions arise. Is it possible that the high test scores achieved by Japanese students can be attributed to some of the differences in educational systems? What are some of the major differences in the two systems? Which differences affect students' performance in the area of mathematics? Which principal difference can easily identified and isolated for study? Does the "whole-class method," i.e., the teaching method typically used in Japan which makes use of open-ended problems, engage students more than using the traditional methods utilized in the U.S. without sacrificing learning?

Hypothesis Statement

Students exposed to the Japanese method of whole-class teaching are more successful at solving open-ended problems than students solely exposed to the American method.

Definition of Terms

For the purpose of this study, "whole-class teaching" will be defined as the teaching style primarily used in Japan, in which instructors guide their students through a series of open-ended problems.

The "American method" will be defined as the teaching method in which the teacher demonstrates how to solve a particular problem type and the students spend most of the remaining class period practicing the skill (TIMSS, 1997).

"Class period" will be defined as the time the teacher and students spend together.

“Lesson” will be defined as the activity or type of instruction used by a teacher. 3

“Open-ended problems” will be defined as problems that foster or develop students’ critical thinking skills through discussions between a student and his or her teacher. These problems may have more than one acceptable solution.

“Seatwork” will be defined as the time in class when students work at their desks.

“Tracking” will be defined as the act of placing students into particular classes according to their ability.

“Base test” will be defined as the test used to compare the classes in this study.

“Open-ended test” will be defined as the test used to determine a student’s ability to solve open-ended problems.

Limitations and Delimitations

A limitation of my study is the small size of my sample. Another limitation is the fact that there are many variables that can affect student learning. While TIMMS identified the common characteristics of teaching styles of Japan and the U.S., not all teachers from the same country use identical teaching methods.

REVIEW OF LITERATURE**The American Method**

The “American method” of teaching is the most commonly used method in the United States for teaching mathematics. William Welch, a researcher, describes a typical lesson from a U.S. classroom in the following way:

First, answers are given for the previous day’s assignment. A brief explanation, sometimes none at all, was given of the new material, and problems were assigned for the next day. The remainder of the class was devoted to students working independently on the homework while the teacher moved about the room answering questions. The most noticeable thing about math classes was the repetition of this routine (Welch, 1978).

Most of the emphasis of a lesson taught in the U.S. was on teaching computational procedures, like long division. Welch’s report found that little emphasis was placed on helping the students develop conceptual ideas. U.S. teachers typically do not help students make connections with procedures and concepts. Another disheartening fact is that 96 percent of the time, students were working on material they had already been shown how to do. The data suggests that students have trouble applying their mathematical skills to new problems. (Welch, 1978). Too often, teachers tell students how to solve problems, instead of asking them to solve complex problems on their own. Traditional teaching in the U.S. does not allow students to solve problems on their own or make discoveries of their own.

Seventeen years later, a similar study conducted by Stevenson and Lee in 1995

drew similar conclusions. In the U.S., children are taught a lesson, given a few 5 examples and then asked to work out several similar problems on their own or in small groups (Stevenson and Lee, 1995). As the class continues, the students occasionally ask for help from their peers or the instructor. For the most part, the students work independently. They have opportunities to learn from the teacher, but in this method the teacher needs to circulate around the room. The instructor must monitor behavior, keep all students on task, and answer questions. The amount of contact with individual students is small. Stevenson and Lee (1995) also found that American students worked individually 47 percent of the time in class, and Japanese children worked individually 28 percent of the time.

The Whole-Class Teaching Method

The whole class method of teaching is predominantly used in Japan to teach mathematics (Stevenson and Lee 1995). In Japanese classrooms, there are more students and less group work. Stevenson and Lee (1995) suggest that children who are taught using the whole-class method derive greater benefit because they have more opportunities to work directly with their teachers. In their study of Japanese mathematics classes, Stevenson and Lee (1995) noticed that teachers relied almost solely on the whole-class method of teaching.

According to Stevenson and Lee, one benefit of whole-class teaching is that "this approach gives the largest number of students the greatest amount of their teachers' time" (Stevenson and Lee, 1995).

Stevenson and Lee (1995) described the typical Japanese class in this way:

In Japan, the teacher may begin instruction by presenting a word problem⁶ and asking the students to discuss the meaning of the problem. The students are then given time to think about how they would go about solving the problem and are asked to write down their solutions. After this, the teacher asks several students to write their approaches on the board and to explain their answers. The students selected are ones who have proposed a type of response that the teacher wants to address and discuss with the whole-class. Before doing this, however, the teacher calls on other students to evaluate the relevance and accuracy of what the first students have reported.

Japanese teachers have successfully used the whole-class approach despite class sizes of forty children of differing mathematical abilities. Stevenson and Lee (1995) found that Japanese teachers have chosen to keep class sizes large, which enables them to teach fewer classes in a day. This schedule allows them to have more preparation time. The increased preparation time means they have more time to develop their lessons and confer with other teachers. Another benefit to the large class size is that there are more diverse answers to each problem.

According to Stevenson and Lee (1995) critics of the whole-class teaching method say that if there are too many children in the class, some children may become inattentive. However, in contrast to this criticism, Stevenson and Lee found that students of all abilities remained alert in class. Each student, despite his or her ability, knew he or she might be called on to participate. A student could be called on at any time for a comment. Students also felt free to work on the solution knowing there is no wrong

approach. Students did not practice the skill the teacher just showed them, but rather 7
they tried to apply all the skills they had learned from previous lessons.

Stevenson and Lee (1995) also noted that when using open-ended problems, Japanese teachers give the students ample time to solve them. As they do this, the teachers circulate throughout the room offering advice and making corrections. In the U.S., teachers tend to use this work time to catch up on paperwork and correct tests. In their study, Stevenson and Lee (1995) reported that students in Japan receive feedback regarding their individual work over 70 percent of the time, but American students receive feedback less than 50 percent of the time.

Open-ended Problems

An integral part of whole-class teaching is the use of open-ended problems. Although this approach is not standard in the U.S., open-ended problems are the main type of problems used in mathematics lessons in Japan. Open-ended problems are used to encourage the curiosity of the students thereby getting them to attempt to solve the problems, rather than applying a single learned concept by memorization. The goal of using open-ended problems is to develop the problem-solving abilities of students (Nobuhiko Nohda 1995).

When using open-ended problems, the instructor identifies a problem to be solved and the students work on finding the solution. The students approach the problem by applying previously learned mathematical concepts. Nohda (1995) states, "This approach has been shown to be most effective in fostering both students' mathematical thinking and students' motivation towards learning mathematics." Nohda (1995) lists several

conditions that should be met when selecting open-ended problems. The conditions are 8 as follows:

- a) Problems that contain subject matter with which the students are familiar.
- b) Interesting content that students would like to investigate.
- c) A feeling of necessity or need to solve.
- d) The merits of the problem could be understood after solving.
- e) Possible to be solved by the students' knowledge at hand.
- f) Students have a feeling of success after solving.
- g) Students would be eager to solve advanced problem
- h) Flexible enough so that changes can be made according to students' interests and ability.

Overview of the TIMSS Findings

The Third International Mathematics and Science Study (TIMSS) is the largest international study of mathematics and science education. It compared results of teaching methods in 41 nations. It evaluated 500,000 students in fourth, eighth, and twelfth grades. TIMSS ranked all the participating nations by computing an average score from the nation's students aggregate test scores. The students were randomly selected for the study. Students with special needs and disabilities were excluded from the test. Therefore, TIMSS cannot be used to draw any conclusions regarding students with disabilities.

In addition to the basic testing of the 41 nations selected, TIMSS chose three

countries, United States, Japan and Germany, for deeper analysis. These countries were 9 chosen specifically because they were recognized as economic powers.

TIMSS contains five parts: (1) student assessment, (2) student and teacher questionnaires, (3) curriculum analyses, (4) videotapes of classroom instruction, and (5) case studies for policy topics.

The student assessment analyzes six specific content areas:

1. Algebra - patterns, relations, expressions, equations
2. Data Representation, Analysis, and Probability - representation and analysis of data using charts and graphs involving uncertainty and probability
3. Fractions and Number Sense - fractions, decimal, percentages, estimation and rounding
4. Geometry - visualization and properties of geometric figures, including symmetry, congruence, and similarity
5. Measurement - units of length, weight, time, area, volume, and interpretation of measurement scales
6. Proportionality - proportionality and ratios

On the survey, students answered questions about their study habits and beliefs about mathematics. The teachers answered questions on their beliefs about mathematics and on teaching practices. As part of the in-depth analysis of German, Japanese, and American methods, the teachers from these three countries were filmed teaching a typical lesson. The videos helped compare the teaching techniques and quality of instruction.

The U.S. students performed slightly below the international average of all 41 countries tested in mathematics. Twenty of the participating countries outperformed the

U.S. The scores from students in thirteen countries were not significantly different 10 from the U.S. The U.S. students outperformed the students from seven countries. The U.S. students scored much lower than the Japanese students in mathematics. The students who were in the 95th percentile of all U.S. students scored significantly below the students who scored at the 75th percentile of all Japanese students. Only five percent of U.S. students scored at the 90th percentile or higher on the mathematics test (TIMMS, 1997). By comparison 32 percent of Japanese students scored at the 90th percentile or higher. In three of the six content areas - Algebra; Data Representation, Analysis and Probability; and Fractions and Number Sense - students from the U.S. scored at about the international average. U.S. students scored below the international average in the other three content areas of Geometry, Measurement, and Proportionality.

An important distinction to be made is that the U.S. and Japan differ in the manner in which they make decisions regarding curriculum (TIMMS, 1997). Japan makes these decisions at the national level. The Japanese National Ministry of Education has specified one set of curriculum guidelines for the whole country. These guidelines determine the topics of study at each level of mathematics education. In the U.S., most school districts determine their own curricula.

In Japan, the National Ministry of Education has approved textbooks published by only six commercial publishers (TIMMS, 1997). These textbooks are very similar to each other because of the strict national guidelines that each publisher must follow. Only minor modifications can be made to the national guidelines by the local school boards.

In the U.S., some of the textbook manufacturers choose to publish books that contain standards adopted by only three states. California, Texas and Florida have passed

laws that school textbooks must contain certain standards. The publishers produce 11 books that contain topics demanded from each of the three states (TIMMS, 1997).

Therefore, textbooks usually contain far more topics and material than can be learned in one year. Due to this fact, the mathematics curriculum used by the typical U.S. district is less focused than Japan's mathematics curriculum.

Teachers from the U.S. used textbooks in almost half of the lessons observed by TIMSS. In Japan, teachers used a textbook in only two percent of their lessons. According to TIMMS (1997) American teachers cover more topics per class than their Japanese counterparts. U.S. teachers taught 1.9 topics per lesson compared to 1.3 topics taught per lesson in Japanese schools. U.S. textbooks were more likely to include lessons dealing with arithmetic operations, whereas Japanese textbooks dealt with algebra and geometry.

It is generally believed that the Japanese students spend far more days in school per year than American students. This myth was dispelled by TIMSS. TIMSS surveyed the teachers on the amount of time they reportedly spent teaching their classes in a school year. TIMSS discovered that U.S. students spend far more time in the classroom studying mathematics. In the U.S. the typical eighth grade students are in mathematics class 143 hours per year. In Japan students are in class 117 hours per year (TIMMS, 1997). The length of time in class does not seem to be the key component in this difference of scores achieved by the U.S. and Japan. In addition, TIMSS reported that Japanese students go to school 40 more days per year than U.S. students. Most of these forty days are spent in Saturday sessions, which are designated for extracurricular activities. With respect to homework, American students spend more time doing

homework than Japanese students.

12

Over 60 percent of teachers in the U.S. surveyed by TIMSS stated that the primary goal of their mathematics lessons was for the students to gain a particular computational skill. Only 27 percent of Japanese teachers stated this as their primary goal. Most teachers (71 percent) in Japan reported that their goal when tackling a mathematics lesson was to have the students use mathematical thinking, such as exploring, developing concepts, or generating multiple solutions to problems. The type of work students were asked to do also shows that the U.S. teachers wanted their students to memorize skills rather than understand mathematics. Since the goals are different the lessons used to teach mathematics are also different. TIMSS described the steps in a typical Japanese mathematics lesson:

- Teacher poses a complex thought-provoking problem.
- Students struggle with the problem.
- Various students present ideas or solutions to the class.
- Class discusses the various solution methods.
- The teacher summarizes the class' conclusions.
- Students practice similar problems.

The following are the steps in a typical U.S. mathematics lesson:

- Teacher instructs students in a concept or skill.
- Teacher solves example with class.
- Students practice on their own while the teacher assists individual students.

In the U.S. students spent 96 percent of seatwork time on routine procedures. In

Japan only forty-one percent of seat work time was spent on routine problems. 13

Japanese students spent far more time thinking mathematically than the U.S. students. Forty-four percent of seatwork time in Japan was spent working on solutions or procedures that required students to think and reason. In the U.S., students spent less than 1 percent of their time developing thinking and reasoning skills (TIMSS, 1997).

In the U.S., mathematical topics were usually just stated to students, whereas in Japan these topics were developed. U.S. teachers stated topics 78 percent of the time and developed topics 22 percent of the time. In Japan, topics were stated only 17 percent of the time and developed 83 percent of the time. In addition, U.S. teachers asked their students to practice computational skills, while Japanese teachers asked their students to analyze problems and explain their reasoning in most classes. While lesson planning, routine procedures, seatwork, and the topics covered by teachers may not be the only differences which contribute to the different TIMSS scores, these trends in teaching do show that U.S. teachers feel it is important for students to learn mathematical skills rather than problem-solving techniques.

Some additional findings by TIMSS also show that the Japanese classroom facilitates learning more than the U.S. classroom. Nearly 25 percent of the U.S. teachers observed by TIMSS experienced outside interruptions. These included announcements on the loudspeaker and visitors at the door. The TIMSS study indicated that the Japanese teacher almost never experienced an outside interruption. Furthermore, U.S. classroom had more internal interruptions. Twenty-three percent of U.S. classes experienced occurrences such as discipline problems or discussions of topics not relating to mathematics. These in-class interruptions occurred in only nine percent of Japanese

TIMSS also judged the content of mathematics classes taught in the U.S. and in Japan. Experts ranked the quality of mathematical lessons using high, medium, and low mathematical content scores. A lesson with high mathematical content includes topics that involve mathematical reasoning. A lesson with medium mathematical content includes topics requiring memorization and computation. A lesson with low mathematical content includes only simple arithmetic computation. Eighty-seven percent of the U.S. lessons were determined as having low mathematical content, while in Japan, this occurred 13 percent of the time. Thirteen percent of U.S. lessons and fifty-seven percent of the Japanese lessons were determined to have medium mathematical content. None of the lessons observed in the U.S. were determined to have high mathematical content, yet in Japan 30 percent of the lessons contained high mathematical content.

In addition to these areas, TIMSS studied the part teachers play in mathematics education. The length of teacher training did not seem to be a problem since almost half of the mathematics teachers in the U.S. held Masters' degrees. Only four other countries in the survey had a higher proportion of teachers with Masters' degrees. TIMSS reported that few of the teachers in Japan earned more than a Bachelors' degree. Although U.S. teachers receive more training than Japanese teachers, the quality of training is quite different. In both countries, people studying to become teachers take a combination of education and mathematics classes. New teachers in Japan receive intensive mentoring during their first year on the job and their relationships with their mentors continue throughout their professional lives. Generally, U.S. teachers do not experience this degree of mentoring during the first year of teaching; few teachers experience any

Despite the fact that teacher training did not seem to be a factor in the test results, the manner in which teachers spent their school day did appear to be a factor. Teachers in the U.S. reported teaching an average of 26 periods per week. In Japan, teachers taught an average of 16 periods per week. This class load of 16 periods per week gave Japanese teachers more time to confer with each other and seek advice (TIMSS, 1997).

The TIMSS study revealed some interesting points regarding the lives of students. Most U.S. schools track their mathematics students by ability. No ability tracking occurs in Japan until 9th grade. U.S. mathematics teachers assign far more homework than their Japanese counterparts. Eighty-six percent of the U.S. teachers give homework three to five times per week. In Japan, only 21 percent of teachers assign homework that frequently. TIMSS also reported that heavy television viewing is as common by Japanese children as it is in the U.S.

In summary, TIMSS highlighted a list of findings from its study. These findings shed light on some of the differences in mathematics teaching in Japan and the U.S. The following is the list of these findings:

- What American teachers teach in eighth-grade mathematics, most teachers from other countries teach in the seventh grade.
- The content of U.S. eighth-grade mathematics lessons requires less high-level thought than classes in Japan.
- The typical goal of a U.S. eighth-grade mathematics teacher is to teach students how to do something. The typical goal of a Japanese teacher is to help students understand mathematical concepts.

- Unlike U.S. teachers, new Japanese teachers receive long-term structured apprenticeship in their profession. 16
- In the U.S. eighth-grade students are divided into different classrooms according to ability. In Japan no ability grouping is practiced.
- U.S. eighth-graders have more hours of instruction in mathematics than Japanese eighth-graders.
- U.S. students do as much or more homework Japanese students.
- Japanese students watch as much television as U.S. students.

CHAPTER III

DISCUSSION AND FINDINGS

Experiment Plan

I conducted the experiment by changing an existing lesson by incorporating the techniques used in Japanese classrooms. I had a control group that was taught using the methods traditionally used in the U.S. Another teacher in the mathematics department taught this class. I taught the experimental group using the Japanese method. Lesson plans have been included in the Appendix. Both classes received a base test to compare mathematical ability (Appendix E). Following the format for teaching in Japan, I used an open-ended question to stimulate the students. My goal was that the students would discover the formula for area of a triangle. After the lesson I gave the students two tests. One test was designed to see if the lesson was a success, and the other, the open-ended test, was designed to determine how well the students could solve open-ended questions (Appendix D). I also surveyed the students to determine their opinions of mathematics teaching.

Discussion of the American Method

A veteran member of the mathematics department at Menahga High School taught the control group. I observed her lesson while she taught. She taught in my classroom during my prep hour, so my presence was not unusual.

She started class by announcing that she was going to teach the students about the area of a triangle. The instructor put the formula ($a=1/2bh$) on the board. She told the

students what each variable stood for. The instructor then told the students they would do three examples, and that they should jot them into their notes.

To start, she drew a right triangle with a base of five units and a height of six units. She labeled the base and height. She then substituted the values into the formula and calculated 15 square units. She asked the students if they understood the process. Two students claimed they understood. The rest of the students sat quietly. As she worked through the rest of the examples she reminded the students that the answer must be some amount of square units. She also called on students to comment on the solutions of the remaining examples. Most students paid attention and took detailed notes. Then she handed out a quiz to test their understanding of the triangle area lesson. As was permitted, some students referred to their notes to work on the test. The students averaged 4.3 questions correct out of six, with three students receiving perfect scores. One student did not solve any of the problems correctly.

Discussion of the Japanese Method

In the experimental class, I began the triangle area lesson by briefly reviewing the previously learned method for computing the area of rectangular figures. All of the students remembered the formula for calculating the area of a rectangle. Next, I gave the students some problems to solve. They were four area problems: two involved rectangles, and two involved large polygons made up of joined rectangles. The students realized that to find the area of a non-rectangular shape, they could insert lines on the shape to form smaller rectangles. Then, they found the sum of the area of the rectangles. Not all students remembered that area could simply be added. At the end of this review

worksheet we discussed all the solutions as a group. Not every student solved the 19 problems in the same way. One problem could be separated into three rectangles, or just two rectangles. However, either approach resulted in the same answer and was equally appropriate. The primary focus of this review was for the students to discover that areas of portions of a figure could be added.

To start the new activity, I drew a right triangle on the board with a height of ten units and a base of six units. The students drew the triangle on a piece of graph paper. I asked them to calculate the area of this triangle. Most students waited for me to provide a formula. I told them that instead of my providing them with the formula into which they would then plug numbers, they should attempt to find the area on their own. One student thought he knew the formula for area of a triangle, and spent his time plugging the numbers into made-up formulas. The other students tried to solve the problem. Four students drew a rectangle around the figure and calculated the area to be 60 square units. Then they took $\frac{1}{2}$ of the area of the rectangle and determined the area of the triangle to be 30 square units. Individually, I asked the four students to construct their arguments for using half the area of the rectangle as the area of the triangle.

When the time came to discuss the solution as a group, three students were ready to explain why it was mathematically correct to cut the rectangle in half. I asked the fourth student to explain her solution to get the discussion started. She noticed that inside the triangle there were several complete unit squares and some portions of units square running along the diagonal of the triangle. These portions could be paired up to form complete unit squares. She found that they totaled 30 square units. This solution was not the desired solution, but it was creative.

Next, I asked a student who had found the correct solution to explain his 20

method. He said he drew a rectangle around the triangle with a length of ten units and width of six units. He said the area was 60 square units. Then he told the class that he cut the rectangle in half and got 30 square units for the answer. He could not, however, explain why it was mathematically valid to cut the rectangle into two congruent triangles. Another student then raised her hand and asked to explain why this was correct.

She noticed that the two triangles formed in the rectangle had the same dimensions, and thus should have the same area. She stated that the two triangles had the same base and height, and also the hypotenuse should be the same, since they were the same segment. She concluded that each triangle had half the area of the rectangle.

Another student added that he thought the triangles also had the same angle measurements. As a group, the class decided this was an adequate method. Next, I had the students attempt to find the area of an acute triangle:

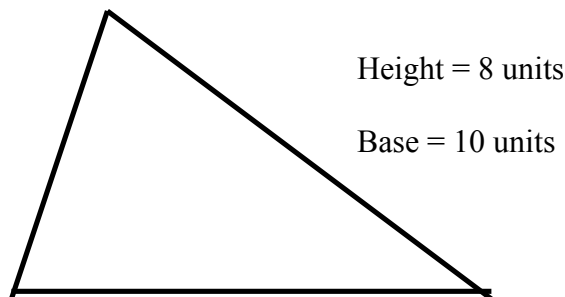


Figure 1.

I drew an acute triangle (Figure 1) on the board. I told the students the height was eight units and the base was ten units. Again, the students copied the triangle onto their graph paper. I then asked them to calculate the area of the triangle. The students worked for about six minutes this time. This time, all students drew a rectangle around the

triangle. Most found the area of the rectangle and then took half to find the area of the 21 triangle. As I walked around the room, I asked the students finishing early to construct an argument to show why it is again possible to take half of the rectangle. When all the students showed signs of finishing I asked for volunteers to provide an answer to the problem.

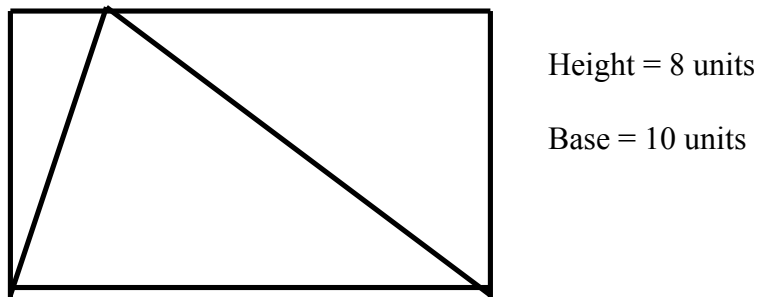


Figure 2.

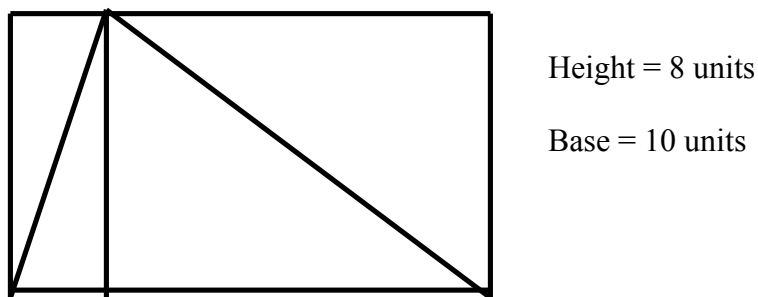


Figure 3.

Most students were eager to discuss their solution. I called on a student who did not comment on the first problem. He came to the board and drew a rectangle around the triangle (Figure 2). He explained the process well and then sat down. I then asked the students if the triangle was really half of the rectangle. Another student thought the triangle was half of the rectangle and offered an explanation. She also came to the board. She altered the existing drawing by drawing a segment from the top vertex of the triangle

perpendicular to the base (figure 3). Now there were four triangles. She said that the 22 two small triangles were congruent and the remaining large triangles were also congruent. Thus the area of the original triangle was again half of the rectangle. The rest of the class agreed with her argument.

After the class discussed these two problems, I gave them the triangle test. The class got an average of 4.8 questions correct out of 6. The obtuse triangle caused trouble for my students. Not one correctly found the area of the obtuse triangle. They sketched a rectangle around the triangle. In this situation though, the length and width of the rectangle are not the same as the base and height of the triangle. They all got answers that were too large.

The goal of the lesson was for the students to generate the formula for the area of a triangle. Although none of the students generated the formula, they could calculate the area for acute and right triangles. Despite being unable to find the formula, the lesson was a success. The students successfully worked independently and found creative solutions to the problem. During the discussion the students helped each other learn. Students were not afraid to solve these problems. They all worked diligently on their solutions. In the survey, 60 percent of my students reported they preferred to teach themselves. One student said, "I like to figure it out myself because I feel I accomplished something." The beauty of the Japanese method is that after the students attempt problems and discuss them as a group, I have the opportunity to wrap up the discussion and emphasize the best solution.

Discussion of Solutions to an Open-ended Problem

One of the open-ended problems the students were asked to attempt was to compute the sum of $1 + 2 + 3 + 4 + \dots + 97 + 98 + 99 + 100$. Most students reported that this problem took about ten minutes to complete. Of the ten students six initially tried to add up all 100 integers. Four tried to find a short cut right away. These same four students came up with a shortcut. Three of the students noticed that $1 + 99 = 100$ and $2 + 98 = 100$. I picked one of these students to explain his work. The student stated, “I noticed $1 + 99 = 100$ and $2 + 98 = 100$. There are 49 pairs of 100, since 50 doesn’t have a pair and neither does 100. This makes the total is $4900 + 50 + 100 = 5050$.” While he talked some students paid attention, and some did not. I should have asked him come to the board and demanded the rest quietly pay attention. Another student added the first and last term, then added the second term and the second to last term and so on. He realized that there were 50 pairs of numbers, each totaling 101. He didn’t want to discuss his solution in front of the class. On his paper he drew an arc from 1 to 100 and from 2 to 99 and so on. I talked with the class about the solution, most liked the first solution better but agreed they were both valid shortcuts. The students were not used to discussing solutions in front of the whole group.

I selected this problem because in my Analysis class we were discussing sums of arithmetic sequences. I put the same problem on the board in that class of 16 upper-level mathematics students (11th grade). I asked the group to find a shortcut. After ten minutes only 1 student found a shortcut. He, like the eighth grader earlier in the day, noticed that $1 + 99 = 100$, $2 + 98 = 100$, and so on and explained his solution in a similar manner as

the eighth grader earlier that day. The students in my upper-level class were waiting 24 for the solution. They have become used to teachers handing them equations and methods for solutions. Students in my classes want good thorough notes and then want to practice what they have learned. The American method of teaching has made the students become dependent on the instructor to provide general formulas. They prefer to work with and memorize instructor-provided formulas, rather than to derive a general solution for problems independently. Teachers who teach using the Japanese whole-class method encourage their students to think for themselves and develop individual approaches for solving problems.

Data Analysis

The students from the class taught using the American method had an average score of 9.4 on the base test. The class taught using the Japanese method scored an average of 9 on the base test. These scores show that the two classes have similar mathematical ability. The scores on the open-ended test were 2.7 for the class taught using the American method and 4.8 using the Japanese whole-class method. When comparing the two classes' scores, I computed a p value of 0.01. This means that at the 99% level of confidence, the scores are significantly different. The evidence suggests that the class taught using the Japanese method was better at solving open-ended tests than the class that was taught using the traditional American method and supports my original hypothesis.

CONCLUSION**Interpretation of Results**

Based on the results of this research paper, I believe that U.S. mathematics instructors can learn something from their Japanese counterparts. In the whole-class method, Japanese teachers have found a successful means for enhancing both teaching and the learning of mathematics. Although it is not practical for the U.S. teachers to completely discard their current teaching methods, U.S. instructors should consider modifying their teaching methods to incorporate key aspects of the Japanese whole-class method. The teachers using the open-ended method encourage students to think about mathematical concepts. The teachers do not simply demand that students memorize formulas and practice skills. Students exposed to the whole-class teaching method are more successful at solving open-ended problems than students solely exposed to the American method. In addition, the study shows that the students who were taught using the Japanese method performed as well on the base test as the students who were taught using the American method.

Suggestions for Further Research

As I analyzed the results of my research, I found that there are several variables I would adjust if I were to do further research on this subject. For instance, the last problem in the open-ended test was too complex for eighth grade students. This problem could have included a simpler model. I picked the problem to see if the students could reason mathematically. I could have tested for mathematical reasoning with an easier

Also the sizes of my classes were small. The students who scored well on the open-ended test could have naturally been good problem-solvers. If this study were replicated, the number of students in the study should be much larger. Similarly, it would help to study a greater number of teachers.

The students in the study were tracked into the lower level of mathematics. I would like to see the affects of teaching a class with students of all abilities. A key component of the Japanese classroom is the inclusion of students of all abilities.

I would change other aspects of the experiment if I were to conduct it again. First I would identify two similar groups of students. I would have a series of lessons ready to teach. I would teach the experimental group using the Japanese whole-class method and the control group using the American method. Then I would test the students' ability to solve open-ended problems, just as I did in the original study. However, I would then switch the groups and teach the control group by using Japanese whole-class method and I would test to see if their open-ended problem solving ability increased.

Implications of the Study

I feel there is a better way to teach mathematics than the way I was taught, and the way I first started out teaching. When I started teaching, I felt that I had to try to teach every section from every chapter in the text I was using. This was a difficult task, since the textbooks I used had 13 or 14 chapters in them. At the end of each school year, I felt that I had not taught my students enough material, simply because I hadn't covered every chapter in the book. I realize now that because I was trying to cover so many topics with

my students, they lacked a conceptual understanding of what they learned. When 27
students learn mathematics, they should be given time to absorb the material and
integrate concepts into their generalized approaches to problem solving. Teaching
mathematics using the Japanese whole-class method appeared to provide students time to
more thoroughly understand topics. If students are taught using the Japanese whole-class
method they will not have to try to learn 13 chapters in one year. The students can spend
an appropriate amount of time on fewer topics and master them. If students are mastering
concepts rather than just speeding through lessons, there will be less need for lengthy
time spent reviewing mathematical skills from previous years. Then, the next year of a
student's schooling can be spent solving new problems and learning new concepts.

U.S. mathematics teachers who still use traditional methods should incorporate
aspects of the whole-class method into their curricula. However, some teachers do not
want to alter their teaching style. Many mathematics teachers use teaching methods that
they experienced when they attended school. Some people can easily learn from the
methods traditionally used in the U.S. Most of the mathematics teachers today enjoyed
mathematics as students. Many mathematics teachers had no trouble learning
mathematics. When they sat in mathematics classes and watched the teacher give notes
for most of the class, they were able to stay interested in the lesson. For them, the
traditional American method was an adequate way to learn mathematics. When they
became teachers, they saw no need to search for an improved method.

Unfortunately, some students who are taught using the American method lose
interest in mathematics. The students who do not enjoy mathematics need more than
notes for an hour a day to learn mathematics. These students would benefit from a

mathematics class that was taught using an alternative method, like the whole-class 28 method. In fact, students at all levels would benefit from mathematics classes taught using the Japanese method.

Some of the new textbooks like *Core Plus*, incorporate the Japanese method into the curriculum. *Core Plus* teaches students to take risks. Teaching from the text makes it easier to implement some activities that require the students to think. Students take pride in their mathematical ability when they have been given a chance to solve problems on their own. They enjoy tackling problems. They enjoy solving new problems on their own.

Last spring, I taught a lesson on discrete mathematics using the *Core Plus* textbook. This textbook is designed to lead the students through a series of thought-provoking problems. After the students attempt to solve each problem, the class then, with the aid of the instructor, discusses the solutions discovered by the students. On the first day of the lesson, I assigned a problem to my class, and had them work on it in small groups. Several students asked me if I forgot to go over the notes first. They wanted to know how to do the problem before they started. This mentality came from their experience in their mathematics classes.

Open-ended problems are not commonly found in textbooks. They are also difficult to design. Summer classes, like the *Math Activities* classes offered at Bemidji State University and sessions at MCTM conferences, are a good start, but more can be done. For example, mathematics teachers could get together once a year to share some of their best open ended activities.

I will continue to search for open-ended problems to challenge my students.

There are topics that I currently cover that could be taught using open-ended problems. 29

For instance, rather than tell my students the formula for calculating the number of degrees in a polygon, I could ask the students to come up with the formula. Similarly the formula for calculating the number of diagonals of a polygon lends itself to open-ended problems. In fact, several topics covered in geometry could be taught using open-ended problems.

Letting the students try to discover mathematical concepts makes the concepts more meaningful to them. The concepts lack meaning when they are simply stated by teachers. Even if students don't make much progress towards a solution when working on a problem, they will have a better understanding of the problem and a stronger desire to hear the solution. The whole-class method builds anticipation. It appeals to the inquisitive nature of children.

Results from class taught using the American Method.

Student	Pre-test	Triangle test	Open-ended test
# of questions	12	6	11
1	7	3	0
2	7	0	2
3	8	3	2
4	9	5	2
5	7	5	2
6	10	4	3
7	11	5	3
8	12	6	3
9	12	6	3
10	11	6	7
average	9.4	4.3	2.7

Results from class taught using the Japanese Method.

<u>Student</u>	<u>Pre-test</u>	<u>Triangle test</u>	<u>Open-ended test</u>
# of questions	12	6	11
1	7	5	4
2	8	5	4
3	8	5	5
4	9	4	2
5	9	4	4
6	9	5	6
7	9	5	5
8	10	5	5
9	10	5	6
10	11	5	7
average	9	4.8	4.8

Lesson Plan for Control Group

- I. State and explain the formula for area of a triangle ($a = \frac{1}{2} bh$. a = area, b = base and h = height)
- II. Discuss Example 1.
Example 1: Calculate the area of a right triangle with a base of 5 units and height of 6 units.
- III. Discuss Example 2.
Example 2: Calculate the area of an acute triangle with a base of 8 units and height of 6 units.
- IV. Discuss Example 3.
Example 3: Calculate the area of an obtuse triangle with a base of 10 units and height of 3 units.
- V. Hand out triangle test.

Lesson Plan for Experimental Group

- I. Review the method for computing the area of a rectangle.
- II. Hand out rectangle work sheet and discuss.
- III. Ask the students to compute the area of a right triangle with a height of 10 units and a base of 6 units.
- IV. Discuss solutions as a class.
- V. Comment on solutions.
- VI. Ask the students to compute the area of an acute triangle with a height of 8 units and a base of 6 units.
- VII. Discuss solutions as a class.
- VIII. Comment on solutions.
- IX. Hand out triangle test.

Survey

Describe your typical hour in math class this year.

List some qualities of a good math class you have had.

List some qualities of a good math teacher.

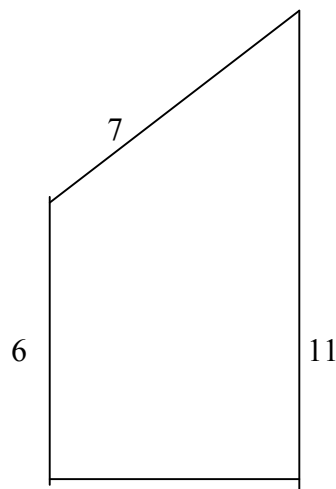
List some qualities of a weak math class.

Which do you prefer, when the teacher shows you how to do each step of a problem, or when you figure it out on your own? Explain.

Open-Ended Test

1) Find the sum. $1+2+3+\dots+97+98+99+100$

2) Calculate the area this shape.



3) How many 6 inch by 6 inch by 6 inch cubes could fit into this classroom (30 feet by 28 feet by 10 feet)?

4) Plumber A charges \$50 to visit and \$10 for each hour. Plumber B charges \$20 to visit and \$20 for each hour. Which plumber is cheaper?

5) Joe hit 12 homeruns in 21 games. At this rate how many homeruns will he hit in 36 the entire 35 game season?

6) Find and describe the pattern for this sequence of numbers: 2, 5, 8, 11, 14,...

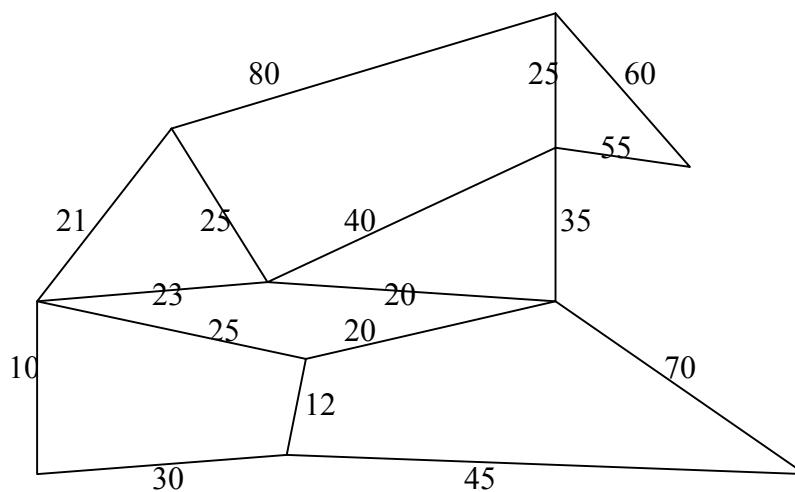
7) Find and describe the pattern for this sequence of numbers: 1, 3, 6, 10, 15, 21,...

8) Find and describe the pattern for this sequence of numbers: 1, 1, 2, 3, 5, 8, 13,...

9) Find and describe the pattern for this sequence of numbers: 4, 7, 13, 25, 49

10) Draw a 3 dimensional shape that has a volume of 30 cubic units. Give all dimensions.

11) The following map shows eleven towns and the length of dirt roads between them. The state would like to pave some of the roads so that a person in any town can get to every other by way of paved roads. Which roads should be paved so that the minimum number of miles has been paved? Color the roads that need to be paved.



Appendix E**Base Test**

- 1) How many students are going on a field trip if $\frac{2}{3}$ of the 720 students are participating in the field trip?
 - a) 240 students
 - b) 360 students
 - c) 480 students
 - d) 500 students

- 2) Jeff bought a CD for \$12.75. The sales tax is 8%. How much did Jeff pay for the CD, including sales tax?
 - a) \$13.55
 - b) \$13.77
 - c) \$22.95
 - d) \$12.81

- 3) 2,450 trees grew when 3,500 were planted. What percent of the planted trees grew?
 - a) 12%
 - b) 55%
 - c) 65%
 - d) 70%

- 4) A town is 5 miles long and 7 miles wide. What is the area of the town?
- a) 12 square miles
 - b) 24 square miles
 - c) 6 square miles
 - d) 35 square miles
- 5) The school cook bought 57 pounds of hamburger at \$1.23 per pound. What was the total bill?
- a) \$70.11
 - b) \$78.23
 - c) \$83.75
 - d) \$88.78
- 6) Jennifer drove 93 miles and used 3.75 gallons of gas. How many miles per gallon did she get?
- a) 18.5 miles per gallon
 - b) 19.6 miles per gallon
 - c) 21.5 miles per gallon
 - d) 24.8 miles per gallon

- 7) Sara can run 1000 yards in 4 minutes. At the same rate how long will it take her to run 2250 yards?
- a) 1.7 minutes
 - b) 7.5 minutes
 - c) 9 minutes
 - d) 12 minutes
- 8) The temperature on Sunday was 17 degrees. The temperature on Monday was -13 degrees. How much colder was the temperature on Monday?
- a) 4 degrees
 - b) 20 degrees
 - c) 24 degrees
 - d) 30 degrees
- 9) The population of a town was 15,000. 10 years later it was 50% larger. What was the new population?
- a) 22,500
 - b) 25,000
 - c) 30,000
 - d) 45,000

10) Sally scored 86 and 95 on her first two tests. What would Sally have to score on the third test to raise her average to 92?

- a) 88
- b) 92
- c) 95
- d) 100

11) A car is 4.5 meters long. The car is _____ millimeters long.

- a) 400 millimeters
- b) 3600 millimeters
- c) 4500 millimeters
- d) 5400 millimeters

12) A store is advertising a 20% discount. Jason bought equipment that regularly sells for \$23.50. How much did Jason actually pay for the equipment?

- a) \$3.50
- b) \$4.70
- c) \$11.75
- d) \$18.80

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