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COOPERATIVE LEARNING IN THE MATHEMATICS CLASSROOM

by

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Student interactions are important which provokes the idea that cooperative learning should be incorporated into classrooms. Research states many benefits for incorporating cooperative learning and are included in the literature review. Cooperative groups were introduced and studied by the author, who ultimately concluded that cooperative learning does have a place in Mathematics classrooms and should be used when the lesson lends itself to a cooperative environment. The study results can be found in this paper following the literature review.

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Chapter 1: Introduction

This paper examines cooperative learning in the classroom, specifically the math classroom. Students come into and out of school not fully understanding some of their classes. Based on both state and federal standards, the majority of schools plan what students need to learn and how students will learn it. Hanna and Yeckel state, "Learning with understanding can be further enhanced by classroom interactions, as students propose mathematical ideas and conjectures, learn to evaluate their own thinking and that of others, and develop mathematical reasoning skills" (as cited in the National Council of Teachers of Mathematics, 2000, p. 21). Many schools adopt curricula that focus specifically on the current standards. It is clear that students interact with one another quite frequently and peer relationships are very important. Some schools incorporate student interactions into standards-based curriculum, which brings cooperative learning into the schools. This paper will look at what cooperative learning is, why it is and should be incorporated into student's daily lives, what researchers have had to say about cooperative learning, cooperative learning's relationship with mathematics, and the author's study on cooperative learning.

Problem Statement

The primary focus of this paper is to examine cooperative learning; however, because math is a subject that people think can only be taught one way, we shall look at the effects of cooperative learning in the math classroom as well. This paper will determine if educators can help students learn math concepts by using methods of cooperative learning in the classroom. This paper will also examine the reasons for implementing cooperative learning and other apparent issues in today's schools as well. We live in a diverse society and students have tendencies to not appreciate others. Acceptance issues are prevalent in schools as well causing difficulty with assigning group tasks and activities. The central question for this paper is: Is cooperative learning be effective in classrooms, specifically math classrooms?

Study Design and Data Collection

Qualitative and quantitative data was collected by conducting a series of group learning activities in two classes and surveying students. One class was taught traditionally with one unit given in a cooperative learning format. The other class was taught non-traditionally and was given multiple units in a cooperative learning format. The format, daily lessons, and data will be provided in Chapter 3 of this paper. Results from the self-evaluations will be presented in a list format. Other results will be listed in charts or tables for easy viewing by the reader and will allow for easier comparisons.

Selection and Description of Site Participants

The researcher selected two classes based on ability to teach the same concepts differently in the same year. Princeton High School utilizes trimesters and so one of the Foundations classes was taught in the second trimester, which runs during the winter and the other Foundations class was taught third trimester, which runs during the spring. Foundations class was selected because the curriculum contains activities ideally designed for cooperative groups and the curriculum contains traditional, non-cooperative lessons as well. Foundations was also selected because the researcher was teaching both trimesters in which students would be taking the second half of their Foundations class; the idea being that with the same teacher, students will have the same overall emphasis on certain topics.

Definitions

The author concluded that cooperative learning is a group of students working together to inquire about and engage in discussion to accomplish a goal. The struggle of arriving at this definition will be discussed more in the Literature Review due to the many definitions encountered through research.

Delimitations

The limitations of this study are:

- The groups studied were pre-algebra only.
- Only 32 students were studied.
- Students were not pre-examined for this study to ensure the groups were of equal ability to start. Caution should be considered when drawing conclusions about the intelligence of one group over the other.

Assumptions

- All students studied where enrolled in Foundations Math Class. The assumption is made that they are all at the same ability level. This does not consider the possibility that some possess ability and lack motivation.

- Studies were conducted with one class in the morning and another class in the afternoon, thus leaving us with the assumption that time of day does not have an impact on learning.
- Studies were conducted between two classes, one during second trimester and the other during third trimester, thus leaving us to assume that time of year does not affect learning.

Chapter 2: Literature Review

Introduction

Ideas about the way that math is effectively taught in schools have been debated for years. Many schools pick up new curricula every five to seven years in an attempt to teach students math concepts using beneficial methods; one such approach is cooperative learning. It is very obvious that since education is the one thing that all people have in common, we find that all people have opinions about the ways that teachers should teach such topics as math. I think many educators have heard the benefits of the cooperative learning philosophy of teaching, but few implement it on a regular basis including the author. The purpose of this literature review is to investigate cooperative learning methods and identify the benefits of teaching students in today's classrooms using cooperative learning. In this literature review, topics such as what cooperative learning is and why it has come into consideration will be discussed. Other topics will be reviewed as well regarding cooperative learning and whether it should be incorporated into the classroom, specifically what implications it has in a mathematics classroom.

The literature review includes the following four dominant topics:

Definition of Cooperative Learning

The Call for Cooperative Learning

Proponents' Position on Cooperative Learning

Relationships Between Cooperative Learning and Mathematics

Definition of Cooperative Learning

While reviewing literature on cooperative learning, I found many definitions of cooperative learning. In this section I will examine a few of the definitions and their similarities and differences. According to Artzt and Newman (1990), cooperative learning is a small group of learners, who work together as a team to solve a problem, complete a task, or accomplish a common goal. Johnson, Johnson, and Johnson Holubec (1994) characterize cooperative learning as working together to accomplish shared goals. These are straightforward descriptions of cooperative learning. Davison and Worsham (1992) characterize cooperative learning as:

> Cooperative learning procedures are designed to engage students actively in the learning process through inquiry and discussion with their peers in small groups. The group work is carefully organized and structured so as to promote the participation and learning of all group members in a cooperatively shared undertaking. (p. xii)

These definitions are similar; the differences are that Davison and Worsham emphasize the idea that cooperative learning is engaging and structured to promote participation among members of the group. The similarities revolve around inquiry, discussion of thought, and a shared goal.

Along with the definition of cooperative learning, this section will also include what cooperative learning looks like in the classroom. It would be beneficial for educators to witness the effectiveness of cooperative learning in classrooms. Since it is almost impossible to have teachers visit another teacher’s room during instruction, an explanation of how a cooperative classroom ideally works will be provided here.

Cooperative learning may be incorporated in many different ways. Since cooperative learning presents itself so diversely, it would be impossible to detail all of the tactics. Johnson, Johnson, and Johnson Holubec (1994) describe normal, daily cooperative classrooms. Typically, "Class members are split into small groups after receiving instruction from the teacher. They then work through the assignment until all group members have successfully understood and completed it" (p. 3). In these groups, students are expected to discuss ideas, help each other uncover connections, complete a task and so on. Students work in groups to clarify their understanding, think and reason together, solve problems, make and test conjectures, and complete other tasks (Davison & Worsham, 1992). From the previous statements, I conclude that cooperative learning presents itself differently in classrooms. However, it is clear that students work together on a given task, help each other clarify concepts, and reason together.

It has become apparent to me while reviewing literature and observing cooperative classrooms that students cannot just be placed into groups for cooperative learning to be effective. According to Johnson, Johnson, and Johnson Holubec (1992), there are five components that need to be considered in order for cooperative learning to be effective (p. 1:11). The five components are positive interdependence, face-to-face promotive interaction, individual accountability/personal responsibility, interpersonal and small group skills, and group processing. More in-depth explanations of these will follow. Kagan (1994) suggests that there are six key components to assure cooperative learning's effectiveness. The six components introduced by Kagan (1994) are teams, cooperative management, will to cooperate, skill to cooperate, basic principles, and structure. Baloche (1998) has examined Johnson, Johnson, and Johnson Holubec's elements for high quality small group cooperation and also focuses on the same five elements.

The differences between Johnson, Johnson, and Johnson Holubec's (1992) elements and Kagan's (1994) are in Kagan's cooperative management, structures, and teams components. These components are not present in Johnson, Johnson, and Johnson Holubec's elements. This does not mean that Johnson, Johnson, and Johnson Holubec do not feel that these are important in cooperative learning, just that they did not feel the need to incorporate them into the elements.

Positive interdependence is stressed in both Kagan's (1994) and Johnson, Johnson, and Johnson Holubec's (1992) elements as the most important aspect of cooperative learning. Positive interdependence is the need for students to perceive that they are linked with their group mates in such a way that they will not succeed unless they all succeed or that they must work together to complete the goal. Positive interdependence and individual accountability are incorporated into the 'basic principles' component of Kagan's elements. This part is when the individual is assessed and held accountable by their group.

The other components of Kagan's (1994) elements and Johnson, Johnson, and Johnson Holubec's (1992) components are lengthy and will not be discussed further. They are crucial in implementing effective cooperative learning; but for the purpose of this paper, indepth discussion is not necessary. The focus will now be on the classroom approaches to cooperative learning.

There are many different approaches to cooperative learning. Davison (2002) states that the common attributes in all the approaches include the following: Common task or learning activity, small-group learning, cooperative behavior, interdependence, and individual accountability. Davison also identifies a range of varying attributes, such as structuring the interdependence, climate, group structures, group leadership and teacher's role. Some of the approaches that have been compared and contrasted to generate the previously stated commonalties and differences are the complex instruction approach, the structural approach, the group investigation approach and the learning together approach.

Johnson, Johnson, and Johnson Holubec (1992) suggest that in order to effectively implement cooperative learning into a classroom, teachers must:

> First, understand what cooperative learning is and how it differs from competitive and individualistic learning. Second, they [teachers] must be confident that using cooperative learning is the most effective thing to do…Third, faculty must realize that simply planting students in discussion groups will not magically produce these outcomes…Fourth, faculty must know that there are many different ways to use cooperative learning…Finally, what is good for students is even better for faculty. (p. 1:11-12)

The above cited researchers of cooperative learning have given these considerations and teachers should account for them when planning to implement cooperative learning into the classroom.

The issue of what the teacher's roles and responsibilities are will be reviewed next. The teacher's role does not just include encouraging students to interact, clarify or adapt their goals, and involve those unlikely to participate; it includes preparing every aspect of cooperative learning. The teacher's role includes initiating group work, presenting guidelines, forming heterogeneous groups, preparing and introducing new material, interacting with small groups, tying ideas together, making assignments of homework or class work, and evaluating student performance (Davison, 1990). Teachers must construct or search to find the right curriculum for the groups. There is considerable time spent preparing for cooperative learning; and throughout this literature review and the study, we will identify whether the positive and negative consequences outweigh time spent preparing for the cooperative classroom.

In this section cooperative learning has been discussed including its components. The common element found in the many definitions of cooperative learning is a group of students working together to inquire about and engage in discussion to accomplish a goal. In this section it has also become apparent that simply organizing groups is not enough; the components of cooperative learning need to be included for maximum effectiveness. Investigations about why to consider cooperative learning in the classroom, what is said about it, and how this applies to math will occur in the following sections.

The Call for Cooperative Learning

This section will identify what researchers discovered about cooperative learning; but first, why did cooperative learning come about? Current issues of educational journals are often focused, either directly or indirectly, on cooperative learning. Cooperative learning has been investigated for many years, but the author's exposure to cooperative learning is fairly recent, which is why the author finds it imperative to discuss. According to the National Council of Teachers of Mathematics (1989), classrooms do not facilitate learning if they have a passive climate. “Proponents of mathematics reform have argued that traditional mathematics instruction, the predominant form of instruction in our nation’s schools, has been unsuccessful in promoting conceptual understanding and application of mathematics to real-life context” (Aslup & Springler, 2003). Johnson (1992) states that the old paradigm is not working because American schools focus on “(a) selecting only the most intelligent students for admission to advanced classes and then (b) inspecting continually to weed out defective students” (p. 1:7). Because of these statements and others, it has become clear that the traditional method for teaching students mathematics is not working.

Smith (1998) emphasizes the faults of the ‘drill and practice structure’ in schools and how people learn from others. Students forget information very quickly unless there are connections made between what they are attempting to learn and their lives; people learn by interacting with others and socializing. The number of people that agree with him, such as Battista (1999), confirm his theory. Battista states, “For most students, school mathematics is an endless sequence of memorizing and forgetting facts and procedures that make little sense to them” (p. 426). Battista (1999) continues to imply that since this is the case, social interactions with others would increase retention of subject matter. The idea is, if students work in groups and learn from each other and with each other, they will be more likely to remember the concepts.

Kagan (1994) recognizes the need for cooperative learning as a global answer to education. He believes that there is a need to incorporate cooperative learning for three major reasons: Socialization practices, economy, and the demographics of society. Socialization practices include the need for students to interact with each other regularly.

> Students today generally do not come to school with the same prosocial values once common; they are not as respectful, caring, helpful or cooperative as they were twenty years ago. The loss of prosocial values and behaviors among students is a result of a number of converging economic social factors. (p. 2:2)

Economic and social factors include family structure and ideals presented to students on television. Kagan emphasizes the need to change the way we look at economics. At one time our nation was an agriculturally based; than it moved to industry, and finally it moved to information-management. The last of the three major reasons for the need to implement cooperative learning is the demographics of society. Kagan states that the 'new majority' is racially diverse. "The new majority does not come to school with the same values and background as did the old majority. They are not responding well to traditional educational structures" (Kagan, 1994, p. 2:7). Sapon-Shevin, Ayres, and Duncan (2002) state that,

> There is increasing cognition that all students, even those currently educated in what appears to be relatively less diverse settings, will need to live and work successfully in diverse, multicultural environments. Cooperative learning can provide students with the skills demanded by our increasingly diverse society. (p. 209)

It has become clear that there is a need for additional teaching methods in schools, including cooperative learning. Now that consideration has been given for why there is a need for cooperative learning, we will focus on what is said about cooperative learning.

In the following section what proponents say about cooperative learning and whether it has shown to be an effective teaching method in schools will be discussed.

Proponents' Position on Cooperative Learning

There are many proponents of cooperative learning. Johnson, Johnson, Johnson Holubec, and Kagan have researched cooperative learning and have very positive ideas about the effects of cooperative learning. According to Kagan (1994), the three most important outcomes of cooperative learning are "(1) academic gains, especially for minority and low achieving students, (2) improved race-relations among students in integrated classrooms, and (3) improved social and affective development among all students" (p. 3:1). Johnson, Johnson, and Johnson Holubec (1994) feel that the major outcomes are student effort to achieve, positive relationships, psychological adjustment/social competence, promotive interaction and positive interdependence. Even though Kagen's outcomes are a bit more general, the similarities between them are clear.

Slavin (1990) reveals that most of the theories supporting cooperative learning fall into two categories: motivational and cognitive. Slavin (1990) states that he and Johnson and Johnson have "found that cooperative learning methods tend to be generally effective in improving intergroup relations, increasing students' acceptance of mainstreamed academically handicapped students and supporting a range of affective concerns" (as cited in Owens, 1995, p. 162). Johnson, Johnson, and Johnson Holubec (1992) do not just acknowledge that cooperative learning helps minorities but extend it to bring different groups together.

> Individuals care more about each other and are more committed to each other's success and well-being when they work together cooperatively [rather] than when they compete to see who is best or work independently from each other…This is true when individuals are homogeneous as well as when individuals differ in intellectual ability, handicapping conditions, ethnic membership, social class, and gender. (p. 22)

Slavin (1990) addresses the issues of whether cooperative learning increases student self-esteem. The idea is that if students feel they are doing a good job learning, their self-esteem will increase. However, Slavin (1990) states, "the evidence concerning cooperative learning and self-esteem is not completely consistent…[in] eleven of the fifteen studies in which the effects of cooperative learning on self-esteem were studied, positive effects on some aspect of self-esteem were found" (p. 44). It is also important to note that Slavin views the effects of student self-esteem on the setting in which they were obtained. "However, these results do suggest that if cooperative learning methods were used over longer periods as a principal instructional methodology, genuine, lasting changes in students' self-esteem might result" (Slavin, p. 44).

Smith, Williams, and Wynn (1995) suggest even more benefits for students. These authors cite Johnson and Johnson (1989) and Slavin (1990) when they state:

> Besides academic achievement, other benefits are associated with cooperative group learning. Some of these benefits are increased retention of the subject matter; increased on-task behavior; increased school attendance; increased student respect for others from various backgrounds; a more positive student

> attitude toward teachers, school, and mathematics; and a greater student self-concept. (as cited in Smith, Williams, & Wynn, 1995, pp. 282-283)

Kohn (1999) states that the individualism of American culture has blinded us to the role that interactions with others play in our coming to understand ideas. Kohn stresses that success in schools is a result of the relationships between students. How they "show and watch, talk and listen, assert and rebut" (p. 153) helps students understand ideas and improves relationships between students. Campell (1996) states, "Researchers who started out with purely individual definitions of what they were trying to teach…arrived at the need for social interaction more through pedagogical trial and error than through theoretical analysis" (as cited in Kohn, 1999, p. 154). Kohn contends that many researchers begin by only wanting to explain the need for students to make sense of mathematical ideas but find themselves seeing the need for 'collaborative dialogue' between students. Johnson & Johnson state, among other specialists, in the following:

> At its best, the practice of having students meet regularly in pairs or small groups not only helps them develop social skills and foster each child's concern about others, but also turns out to be powerfully effective in intellectual terms. This is true for several reasons.
>
> 1. A student struggling to make sense of an idea may understand it better when it is explained by a peer (who only recently figured it out himself) rather than by an adult.
>
> 2. The student who does the explaining can achieve a fuller understanding of the subject matter by having to make it understandable to someone else.

> This is why cooperative learning has been shown to benefit the one giving the explanation at least as much as the one hearing it.
>
> 3. Having a group tackle a task is typically far more efficient than having one person do it alone, since students can exchange information and supplement one another's investigations.
>
> 4. Cooperative learning often leads students to become more motivated to learn; their attitude improves, and that, in turn, facilitates their achievement.
>
> 5. Finally, remember that constructing meaning typically takes place through conflict, and conflict happens when students have the chance to challenge one another in an environment that feels caring and safe. Disagreement doesn't imply an adversarial encounter; it's a "friendly excursion into disequilirium," in the lovely phrase of David and Roger Johnson. (as cited in Kohn, 1999, pp. 154-155)

In the last passage, clarification of the benefits earlier mentioned were given to connect the benefits with the rationale behind the benefits. The benefits listed earlier are just some of the advantages of cooperative learning; not all specialists are proponents of the advantages and certainly most do not limit the advantages to only these.

Now that the advantages and benefits of cooperative learning have been reviewed we can focus on specific advantages of using cooperative learning in the mathematics classroom. The following section will focus on the relationships between cooperative learning and mathematics, whether cooperative learning can be applied in mathematics classrooms effectively and produce positive results or if cooperative learning should be excluded from math classrooms.

Relationships between Cooperative Learning and Mathematics

The author has met some educators of mathematics who think cooperative learning is perfect for some subjects but is not designed for mathematics. This is a common misconception and possibly an excuse to exclude mathematics educators from learning new and more effective methods of instruction. Heaton (2000) discusses some of his concerns and experiences when working with mathematics classes and incorporating cooperative learning:

> I was unsure just what was important to learn. What was there besides rules for students to know? On what conceptual mathematical ideas was this rule based? Why and how did the rule work? There had to be some underlying mathematical meaning for these rules. At the time, these were meanings I did not understand.
>
> In this series of lessons, I was struggling between two different conceptions of mathematical knowledge, loosening my hold on rules and procedures, while searching for some deeper conceptual meaning. Being uncertain was unsettling…Learning the rule to add and divide was not the ultimate goal. There had to be more to this. But what? If reasoning was what I was after…was I to value their reasons even if they supported a wrong solution? (p. 130)

The point that Heaton made is that doing this cooperative approach to learning mathematics requires effort. It is not an easy way for teachers to instruct that involves no thinking on the part of the educator. Teachers often do not fully understand the reasons for the rules and that makes teaching students through cooperative learning difficult. However, the real question is this: What is the purpose of education? According to The National Council of Teachers of Mathematics (2000), "If students are to learn to make conjectures, experiment with various approaches to solving problems, construct mathematical arguments and respond to others' arguments, then creating an environment that fosters these kinds of activities is essential" (p. 18). Therefore, educators must understand the underlying reason and be skilled at facilitating cooperative work. According to Heaton (2000), cooperative learning should not exclude mathematics simply because it is more challenging for teachers to teach this way. Instead, it should be included to facilitate full reasoning and understanding for both students and teachers.

There are other reasons for implementing cooperative learning into the mathematics classrooms. The National Council of Teachers of Mathematics (2000) indicates that truly understanding mathematics while learning it is essential. The National Council of Teachers of Mathematics (2000) cites Hanna and Yeckel, “Learning with understanding can be further enhanced by classroom interactions, as students propose mathematical ideas and conjectures, learn to evaluate their own thinking and that of others, and develop mathematical reasoning skills” (p. 21). By doing the first three activities listed in the previous statement, students develop mathematical reasoning skills, which are essential and one of the main purposes of teaching students mathematics. According to Owens (1995), using cooperative learning in mathematics classrooms will increase student participation and peer support, which will result in a decrease in anxiety. “Mathematics, Davidson argues, is ideally suited to a cooperative learning approach because its problems can persuade one another by the logic of their arguments” (p. 155).

As indicated earlier in ‘The Call for Cooperative Learning’, cooperative learning is an effective way to increase mathematical ability in minority groups. Women and minorities are highly underrepresented in any math field. Since cooperative learning is known to increase mathematical skills for minorities, it needs to be welcomed in the mathematics classroom as an effective instructional strategy.

Slavin (1990) reviewed a study conducted on cooperative learning. The goal of the study was to examine standardized test scores in various subject areas, including language arts, science and math, of students in both cooperative learning groups and control groups where little, if any, group work was included. The study analyzes cooperative learning methods called Student Team Learning Methods. The Student Team Learning Methods include STAD, TGT, TAI, and CIRC. STAD is for Student Teams-Achievement Divisions, TGT is for Teams-Games-Tournament, TAI is for Team Assisted Individualization and CIRC is for Cooperative Integrated Reading and Composition. There are other cooperative learning methods such as jigsaw, group investigations, and learning together; but they were not included in this study. The study of the previously mentioned methods for learning was conducted and compared against control groups (not cooperative classrooms).

> Overall, the effects of cooperative learning on achievement are clearly positive: 49 of the 68 comparisons were positive (72%); only 8 (12%) favored control groups…[the study] reveals that different cooperative learning methods vary widely in achievement effects. (p. 18)

It is clear from the reviewed literature that cooperative learning has shown to be effective in any subject area. On the other hand, it should be noted that most of the research involving cooperative learning in mathematics has been conducted with younger students, mostly students at or below eighth grade. Very few investigations have taken place in a high school setting. Hellinan (1984) states that the most current research focuses on achievement and suggests that other factors need increased attention (as cited in Owens, 1995). These factors include group composition, interactions between groups, and unintended consequences of grouping practices. The suggested group composition focus would include conducting studies of what types of grouping practices work best, how groups can influence one another, and what, if any, are the unintended consequences of grouping students.

Summary

The reviewed literature reflects solid support for using cooperative learning in the mathematics classroom as well as in all classrooms. A common theme throughout the literature reflects that as students work in cooperative groups, they gain a deeper understanding of concepts. As Artzt and Newman (1990) wrote, “In this way, students can talk about the problem under consideration, discuss solution strategies, relate the problem to others that have been solved before, resolve difficulties, and think about the entire problem-solving process” (p. 1). Many research-based organizations have made statements about how best to educate students; many of them support the same reasons for use that The NCTM has stated about student learning. The call for cooperative learning has been ongoing for quite some time. I have discovered some of the suggested benefits of using cooperative learning through research and looked forward to observing the benefits in my classroom. The primary reasons the author chose to consider cooperative learning in the classroom include the following: Cooperative learning

- helps students with societal changes (Kagan, 1994),
- facilitates deeper understanding of the subject (Kagan, 1994),
- helps foster learning in minority groups (Johnson, Johnson, & Johnson Holubec, 1993),
- helps students evaluate their own thinking and the thinking of others (National Council of Teachers of Mathematics, 2000),
- helps students gain an acceptance for one another (Owens, 1995),
- helps students to be actively engaged in their learning (Davison & Worsham, 1992), and
- If students are to learn to make conjectures, experiment with various approaches to solving problems, construct mathematical arguments and respond to others' arguments, then creating an environment that fosters these kinds of activities is essential. (The National Council of Teachers of Mathematics, 2000, p. 18)

The Literature has focused on what cooperative learning is, why it has come about, and what proponents have to say about cooperative learning, and the relationships between cooperative learning and Mathematics. The following section will list the study that was conducted and the results followed by a chapter dedicated to a summary of data and the researchers thoughts and conclusions.

Chapter 3: The Study and Findings

Introduction and Guiding Questions

The research question for this paper is: How can cooperative learning be effective in classrooms, specifically math classrooms? There were two study groups, one involved primarily traditional teaching methods and the other involved primarily cooperative learning activities. This study was completed in Princeton Minnesota at Princeton High School with Foundations (Pre-Algebra) students. This chapter will be dedicated to outlining the study and recording results.

Selection and Description of Participants

The researcher has taught Foundations Math six times and thought teaching a class that she was very familiar with would be the best for the study. Thirty two 9th and 10th grade students participated in the study. Each class contained approximately sixteen students. Group A will be the mostly traditional group with one unit involving cooperative learning and Group B will be the mostly non-traditional group with most lessons involving group activities.

Group A started with 19 students reduced to 16 throughout the trimester due to moves of families and removal by counselors for scheduling reasons. Four students had Individualized Education Plans (IEP), one on a 504-accommodation plan and four flagged by the school as "at-risk" students. Students are flagged as "at-risk" by the school due to family circumstances or medical reasons for identifying them as needing extra attention or to be watched for sudden drops in performance, etc. Students on 504-

accommodation plans are in need of special services but for some reason do not qualify for an IEP.

Group B consisted of 16 students at the beginning of the class, which dropped to 15 near the end of the trimester. Six students on IEP's, one student on a 504-accommodation plan, and four students considered "at-risk".

Lesson Plans for Duration of Study

Group A: Traditional Class with One Unit of Cooperative Groups

Chapter 5 and sections 6-1 were given strictly traditionally. Format each day was as follows:

- Students enter class and 10 minutes was spent on questions from the previous day's homework. Homework was collected and next lesson was presented by teacher. Students took notes on new section via whiteboard or smart board and example problems were done on board by teacher and by students. Class was assigned homework to be completed and brought back at the start of the next class period.

Sections 6-2A, B, & C were given non-traditionally and the format for each day is as follows:

Day 1:

Teacher gave students their groups and talked about roles (Manager, Recorder, Clerk, Facilitator).

6-2A Section in book, see Appendix A.

Students took turns reading until the Consider section. Then student in groups wrote down and thought about answers together. After a couple of minutes, students took turns reading as a class until the Explore section, where students again worked with their group mates. Class read again until the Try It! section and they did that in groups as well. The students were then given a 12 problem assignment. They worked with groups for the remaining class time.

Day 2:

The first 15 minutes were given to the groups to wrap up any remaining questions from the previous day's assignment. Note: Students were not able to complete as homework because they involved protractors that do not leave the classroom and/or do not have the opportunity to complete with group mates. Students then separated and took a quiz (6-2A) on the lesson from the previous day (Average score found in table 5). Students were asked to complete a self-evaluation, see Appendix B, on how they felt the group work went on day one. The results from day one and four are listed in table 1 and 2. Students then moved back into their groups. Students read as a class 6-2B, see Appendix B. Groups worked on Consider, Explore and then started their group work problems which consisted of 12 problems.

Day 3:

The first 10 minutes were given to the groups to wrap up any remaining questions from the previous day's assignments. Students separated and took a quiz (6-2B) on the lesson from the previous day (Average score found in table 5). As a class students read 6-2C, see Appendix A. Groups worked on Consider, Explore and then started their group work problems, which consisted of 12 problems.

Day 4:

Students were given a couple minutes to complete unfinished problems. Students then separated and took quiz (6-2C). Students then regrouped and took test 6-2 as groups. Students were asked to complete the Self-Evaluation (results in table 2) again along with any additional comments they had on group work.

Group A did the rest of the lessons in Chapter 6 in a traditional format along with Chapters 7 and 8.

Data Collection Strategies

The data collected is given in the following tables. The number of students who responded are listed and all responses where given on a scale of 1 to 4 with 4 being they mostly agreed. Later in this chapter, a table comparing overall grades of the two classes studied will be given.

Table 1

Group A Self-Evaluation Given after Day 1

Task	Agree 4	Somewhat Agree 3	Somewhat Disagree 2	Disagree 1	NA
Performed their assigned roles	14	2			
Understood the purpose of the Explore	13	2	1		
Understood the solution to the Explore	11	1	2		
Were able to answer the Consider and Try IT	15	1			
Listed to each others' ideas	12	3	1		
Gave feedback to those who contributed ideas	7	7	1	1	
Stayed on task	13	3			
Assisted in preparing the work that was collected	13	2	1		
Had their assignment from the previous day	14	1	1		
Expressed their ideas to the group	11	3	1	1	
Were willing to compromise when needed	12	2			2
Actively participated in the group	12	1	1	2	

Table 2

Group A Self-Evaluation Given after Day 4

Task	Agree 4	Somewhat Agree 3	Somewhat Disagree 2	Disagree 1	NA
Performed their assigned roles	15				
Understood the purpose of the Explore	12	3			
Understood the solution to the Explore	12	3			
Were able to answer the Consider and Try IT	14				
Listed to each others' ideas	14	1			
Gave feedback to those who contributed ideas	10	4		1	
Stayed on task	13	2			
Assisted in preparing the work that was collected	14	1			
Had their assignment from the previous day	13	2			
Expressed their ideas to the group	9	5		1	
Were willing to compromise when needed	13	3			
Actively participated in the group	12	3			

Comments students made after day 1:

- "I think this is dumb because if someone gets a bad grade we all do and I learn more working by myself",
- "It was really fun. I got most of it done when I was in a group."
- "It went good but I don't like how you took someone's paper by random because if someone decided to not do it, it would effect the groups grade."

- "It was alright but the kids I was with are kinda shy but I thought it was okay being in groups."
- "The group work went good but I liked it better when we didn't work all together because now it just seems like I'm not learning anything all that well, but my group did a good job and understood them."
- "I enjoy the group work because for me it's easier to learn in a group. It's also good cause the whole group actively participated making us all learn.",
- " It was good, I'd rather work alone but its nice to check your answers with your group members. It's easier for me to learn things by myself than to have 3 different people telling me different things. My opinion, don't do it again."
- "yesterday went good. I like working together because I can ask for help when I need it."
- "We all did our parts/jobs. Worked well with each other. Helped out one another. Gave ideas for answers."
- "I think this group work sucks. I don't think I should be graded according to other peoples work. This is just going to bring my grade down anyways, so why do it?"
- "Being groups that we didn't get to choose didn't really work out. We're kinda shy around people that we don't talk to, so it wasn't really working that good."
- "I think that one person could pull all the weight in these groups but it was not a problem in ours everyone stayed on task and this is a better way to learn for people who are struggling."

Comments students made after day 4:

- "Nothing really changed with out group"

- "We shouldn't have tasks (ex. Clerk, Manager), some people can't be a facilitator or a helper."
- "helpful"
- "not fun"

Group B was taught traditionally until section 6-2, similar to Group A. The rest of Chapter 6, Chapter 7, and Chapter 8 were given in the cooperative learning format. The first four days were given identical to Group A, see page 25. Students were given the same self-evaluation after day one and after day four and asked to give comments on the group activities.

Table 3

Group B Self-Evaluation after Day 1

Task	Agree 4	Somewhat Agree 3	Somewhat Disagree 2	Disagree 1	NA
Performed their assigned roles	7	6	1		
Understood the purpose of the Explore	7	5	1	1	
Understood the solution to the Explore	7	5	2		
Were able to answer the Consider and Try IT	8	2	4		
Listed to each others' ideas	6	4	3	1	
Gave feedback to those who contributed ideas	7	5	2		
Stayed on task	6	5	3		
Assisted in preparing the work that was collected	11	1	1	1	
Had their assignment from the previous day	10	4			
Expressed their ideas to the group	8	6			
Were willing to compromise when needed	7	4	2	1	
Actively participated in the group	8	3	3		

Table 4

Group B Self-Evaluation after Day 4

Task	Agree 4	Somewhat Agree 3	Somewhat Disagree 2	Disagree 1	NA
Performed their assigned roles	10	3			
Understood the purpose of the Explore	10	3			
Understood the solution to the Explore	9	3	1		
Were able to answer the Consider and Try IT	10	3			
Listed to each others' ideas	10	3			
Gave feedback to those who contributed ideas	8	5			
Stayed on task	9	3	1		
Assisted in preparing the work that was collected	11	1	1		
Had their assignment from the previous day	11		1	1	
Expressed their ideas to the group	10	2	1		
Were willing to compromise when needed	12		1		
Actively participated in the group	10	3			

Comments made after day 1:

- "I think it went ok but some people don't like to stay on task and they talk allot which prevents them off task and they forget what they are doing, they talk way to much other than what there doing, it would be fun if they did what they were doing and didn't have to always ask me. And they do talk nasty but their boys! They just need to stay on task and it would be better."
- "It was easy, we got work done."

- "I really like doing group work. I like working with other people, then by myself. It think it was a good help."
- "Its good"
- "I think all groups should be disbanded and we all go back to everyone doing their own work."
- "I don't like doing groups, some people didn't want to do the work. I could get a bad grade even if I earned a good grade. I think that the group this is not a good idea."
- "everyone did their part. Some wanted to help themselves for a while and helped the others when they were done. Our group did pretty well. Some didn't understand, but we all got things in the end."
- "I like group work better because we all get more help. Its also funner because we get to talk to each other. I like that one test is the score."
- "I liked doing this group thing because we get help out the other people that don't get it and we get to use teamwork so that was a plus. All together I really enjoyed it!"
- "I liked it. Everyone did there job for the most part but it would be better if Dylan wasn't whining about his grades every second and yea that's it."
- "I love this idea its a lot funner easier to learn every thing having every one near to me helping me step by step every body worked hard. I think its fun."
- "I work a lot better in groups. I like it a lot. 1-4 how cool is travis? 4"
- "The way I feel about the group idea is that it's pretty good. It's better then doing it by yourself."

- "I like doing group work it goes good I think besides sometimes it gets too loud. Its fun, teamwork and I think everyone likes it just some don't like their groups but they can deal with it."

Comments made after day 4:

- "Today and yesterday was good the group idea is good."
- "We did so much better then before then we have in the past. Still weird talks but…"
- I feel today our group really understood what to do and we all stayed on task really good mostly Cole."
- "I think we did better than yesterday because everyone cooperated better than yesterday."
- "Groups are ok today. I feel really smart cause I knew how to do todays math work. Yup yup."
- "I liked the groups. They should stay the same!"
- "I liked the groups we should keep the same groups at least work in groups forever."
- "good"
- "Same as last survey" ["I think all groups should be disbanded and we all go back to everyone doing their own work."]
- "I thought today went good. I'm glad we finished the assignment. I hope we keep doing group work."
- "We need help on some things but otherwise we did really well."

- “People that have different answers and disagreeing and having a different score. I don’t think this is right.”
- “Worked very well. No arguments.”

The table below gives a glimpse of the gradebook. Included in the table are the identical tests and quizzes that both classes took. There are other assignments that were either given to one or the other class in chapters 5, 7, and 8. They are not included providing more direct comparisons among the two classes. Tables 6 and 7 give a complete gradebook scores for both groups.

Table 5

Common Assessments among Study Groups

Group	Test 5-2	Ch5Test	6-1 Test	G6-2a Quiz	G6-2b Quiz	G6-2c Quiz	*Test 6-2	**Ch 6Test	7-1Test	7-2a Quiz	Ch7 Test	8-2Test	***Final Exam
A/traditional	72.80%	61.09%	18.78	6.33	4.67	10.22	13	35.83	16.5	5.39	70.13%	20.89	71.04%
B/cooperative	45.89%	72.14%	20.13	6.38	4.63	10.69	12.06	30.69	15.44	*3.56	48.26%	18.81	61%
		Total pts:	28	8	6	14	20	52	24	8		27	

* Completed independently and scores were averaged with group mates

** Group A completed independently and Group B completed with group mates and one test of the groups was graded.

*** All students took independently

both class did cooperatively

Table 6

Grade Book Group A

Group	Incent	Final	Week	Week	T 8-2	Ch 7 T	7-2A	T7-1	Ch6	week 3	T6-3	GT6-2	GQ6-2	GQ6-2	GQ6-2	T6-1	week 1	T Ch5	T5-3	T5-2	Q5-2A	T5-1	Q5-1A	Extrac	Book N
	03/07/2	03/06/2	03/03/2	03/03/2	02/27/2	02/15/2	02/11/2	02/05/2	01/30/2	01/29/2	01/28/2	01/23/2	01/18/2	01/16/2	01/16/2	01/14/2	01/08/2	12/29/2	12/20/2	12/13/2	12/12/2	12/06/2	12/05/2	12/04/2	12/03/2
	F EX	F EX	DW	DW	TEST	TEST	QUIZ	TEST	TEST	DW	TEST	TEST	QUIZ	QUIZ	QUIZ	TEST	DW	TEST	TEST	TEST	QUIZ	TEST	QUIZ	DW	DW
01	0	11	14	18	19	21	6	16	35	14	19	15	9	5	6	16	13	28	21	16	18	17	11	EX	41
02	0	29	14	14	26	32	3	14	41	14	35	15	9	5	6	20	13	57	20	21	22	23	20	EX	115
03	0	28	10	16	20	27	6	16	44	14	43	14	11	6	8	24	9	57	30	27	24	23	20	EX	63
04	0	20	14	18	24	21	6	16	37	14	21	11	9	4	7	21	14	8	15	21	17	18	17	3	71
05	0	21	13	15	22	19	4	19	37	14	17	9	11	3	5	18	8	37	12	28	22	22	20	EX	84
06	0	19	14	17	24	13	6	10	31	14	32	12	9	4	7	17	14	36	26	21	21	20	16	EX	112
07	8	16	10	14	20	24	7	14	30	12	17	6	12	6	6	19	10	30	18	14	15	17	11	0	15
08	0	28	12	18	24	32	8	22	50	14	40	14	13	6	8	23	11	52	23	20	23	19	18	EX	9
09	0	21	6	8	26	27	5	17	21	10	37	12	9	4	7	20	12	44	24	25	19	20	17	EX	19
10	0	22	10	18	24	27	6	17	39	14	31	17	11	5	7	12	13	53	28	25	24	23	18	EX	99
11	14	19	10	16	0	17	4	19	38	14	23	12	11	6	8	17	14	26	19	18	13	15	15	EX	49
12	0	25	14	16	27	19	6	12	32	14	31	17	11	5	7	17	14	50	30	26	19	18	17	EX	97
13	13	22	8	16	27	17	0	14	43	12	29	14	8	6	8	18	8	36	17	18	17	17	12	EX	105
14	15	23	12	18	24	23	8	21	40	14	35	15	9	5	6	23	14	46	20	28	21	20	16	EX	2
15	15	22	14	16	26	28	4	18	34	14	37	17	11	6	8	23	11	42	29	20	17	22	17	EX	74
16	8	14	8	14	21	21	6	16	26	9	28	15	9	2	0	16	EX	EX	EX	EX	EX	EX	EX	EX	8
Max Sc	18.00	28.00	14.00	18.00	27.00	32.00	8.00	24.00	52.00	14.00	46.00	20.00	14.00	6.00	8.00	28.00	14.00	65.00	34.00	29.00	26.00	23.00	20.00	0.00	100.00
Avg Sc	9.13	19.89	10.72	14.78	20.89	22.44	5.39	16.50	35.83	12.83	30.06	13.00	10.22	4.67	6.33	18.78	11.50	39.71	21.59	21.12	17.56	19.53	15.71	0.00	0.00
Percen	51%	71%	77%	82%	77%	70%	67%	69%	69%	92%	65%	65%	73%	78%	79%	67%	82%	61%	64%	73%	68%	85%	79%		

Table 7

Grade Book Group B

Group B	Incent	Final	TCh8so	T8-3so	DW8-3a	Q8-3as	T8-2	wksh 8	Q8-2b	Q8-2Ap	T8-1So	HW8-1s	G 8-1A	Ch 7 T	GT 7-2
	06/05/2008	06/03/2008	06/02/2008	05/28/2008	05/28/2008	05/27/2008	05/23/2008	05/23/2008	05/22/2008	05/21/2008	05/20/2008	05/20/2008	05/16/2008	05/14/2008	05/13/2008
	F EX	F EX	TEST	TEST	DW	QUIZ	TEST	DW	QUIZ	QUIZ	TEST	DW	QUIZ	TEST	TEST
01	9	24	27	21	6	6	21	6	7	8	25	8	6	30	13
02	0	24	18	14	6	2	24	4	0	8	20	8	7	27	12
03	11	20	27	20	6	6	22	5	8	6	33	7	7	18	12
04	8	18	20	12	6	6	21	6	7	6	17	8	7	21	9
05	0	17	26	15	5	4	17	5	10	8	22	7	7	26	8
06	8	21	28	22	6	4	18	3	6	6	27	8	4	20	9
07	0	0	13	20	0	4	23	5	10	6	18	3	7	24	13
08	0	16	15	11	6	6	18	5	6	8	24	7	6	24	8
09	8	20	21	18	6	4	22	6	6	2	13	2	4	29	9
10	0	11	13	10	6	6	19	6	2	6	30	8	7	20	9
11	0	8	21	17	6	2	12	6	5	6	16	6	6	37	12
12	8	12	9	17	5	6	17	5	6	6	10	6	7	15	13
13	9	17	21	10	0	4	19	6	4	6	17	7	7	21	13
14	10	13	16	17	6	4	16	4	0	8	24	7	4	14	9
15	5	18	23	13	4	0	14	3	0	8	13	5	4	2	8
Max Score:	14.00	29.00	38.00	23.00	6.00	6.00	27.00	6.00	10.00	8.00	36.00	8.00	8.00	43.00	20.00
Avg Score:	7.44	15.93	18.63	14.81	4.94	4.38	18.81	5.06	5.25	6.63	20.00	6.31	6.00	20.75	10.19
Percent:	53%	55%	49%	64%	82%	73%	70%	84%	53%	83%	56%	79%	75%	48%	51%

Group B	GQ7-2c	G7-2	GQ7-2B	week e	GT7-1	7-1ABC	Q7-1C	Q7-2A	7-1A&B	Behavi	6-3ABC	GT6-3	Ch 6 T	GQ6-3B	GQ6-3A	T6-2
	05/12/2008	05/12/2008	05/09/2008	05/09/2008	05/07/2008	05/07/2008	05/07/2008	05/07/2008	05/03/2008	05/02/2008	04/30/2008	04/30/2008	04/30/2008	04/26/2008	04/24/2008	04/23/2008
	QUIZ	DW	QUIZ	DW	TEST	DW	QUIZ	QUIZ	QUIZ	DW	DW	TEST	TEST	QUIZ	QUIZ	TEST
01	7	8	0	25	20	6	16	4	16	25	8	24	31	4	14	10
02	6	8	8	25	20	6	16	6	16	25	8	24	42	4	14	16
03	6	7	8	25	13	3	13	6	16	25	4	33	32	4	13	8
04	5	5	2	24	20	5	16	7	16	24	4	35	34	4	12	20
05	5	8	2	24	20	6	16	6	16	25	8	21	39	8	13	12
06	5	7	2	24	16	5	16	7	15	24	3	35	20	4	12	20
07	10	6	5	25	16	5	16	4	15	25	5	21	41	3	13	20
08	5	8	2	25	16	6	16	6	15	25	7	24	34	4	6	12
09	5	5	2	22	16	5	16	7	15	19	6	21	27	3	13	14
10	6	8	8	25	14	4	7	6	16	20	7	33	23	4	13	8
11	6	8	8	25	16	5	7	6	16	23	8	33	39	4	13	6
12	7	8	2	25	16	4	10	4	16	22	6	35	18	0	9	12
13	7	8	2	25	18	3	7	4	16	22	1	21	32	3	4	8
14	5	5	2	24	13	3	13	5	16	22	2	2	30	3	12	10
15	5	8	2	20	13	3	13	6	16	17	7	24	19	4	14	8
Max Score:	16.00	8.00	8.00	25.00	24.00	6.00	16.00	8.00	16.00	25.00	8.00	46.00	52.00	8.00	14.00	20.00
Avg Score:	5.94	7.19	3.56	24.19	15.44	4.50	12.38	5.63	15.75	23.00	5.69	24.13	30.69	3.81	11.81	12.06
Percent:	37%	90%	45%	97%	64%	75%	77%	70%	98%	92%	71%	52%	59%	48%	84%	60%

Group B	Q6-2D	GQ6-2C	Behavi	6-2BQu	6-2AQu	T6-1	6-1ABC	6-1cda	6-1a	HW Ch	T Ch 5	Behavi	T5-2	5-1A&B
	04/23/2008	04/21/2008	04/18/2008	04/18/2008	04/18/2008	04/14/2008	04/14/2008	04/10/2008	04/09/2008	04/07/2008	04/07/2008	04/07/2008	03/31/2008	03/24/2008
	QUIZ	QUIZ	DW	QUIZ	QUIZ	TEST	DW	QUIZ	QUIZ	DW	TEST	DW	TEST	QUIZ
01	8	12	25	2	8	28	8	6	5	14	32	25	27	30
02	8	12	25	2	8	21	8	6	5	14	25	25	24	50
03	8	10	25	6	8	21	7	2	5	6	19	25	16	17
04	7	11	25	6	8	21	7	5	5	10	27	24	14	39
05	4	10	25	6	8	20	5	6	3	8	32	25	9	35
06	7	11	24	6	8	19	4	6	5	6	22	25	14	26
07	7	11	25	4	6	22	0	5	6	0	34	25	22	10
08	4	10	24	6	8	23	5	6	5	10	30	24	15	8
09	6	12	22	2	8	21	8	6	4	14	24	21	16	11
10	1	9	21	6	2	13	8	2	4	14	25	25	8	41
11	3	9	25	6	2	13	9	6	4	14	22	25	10	11
12	4	12	25	6	8	27	5	4	5	14	22	24	9	11
13	2	9	24	6	2	21	5	4	4	6	25	25	19	13
14	8	12	24	2	8	21	5	6	4	8	20	25	6	0
15	1	9	19	6	2	9	6	6	4	8	17	22	10	22
Max Score:	8.00	14.00	25.00	6.00	8.00	28.00	8.00	6.00	6.00	14.00	35.00	25.00	29.00	20.00
Avg Score:	5.19	10.69	23.94	4.63	6.38	20.13	6.00	5.13	4.50	10.00	25.25	24.38	15.31	22.87
Percent:	65%	76%	96%	77%	80%	72%	75%	86%	75%	71%	72%	98%	53%	114%

The results of the study provoke many questions about cooperative learning and how it affected these groups. In the following chapter, the author will discuss her thoughts on the results and give her interpretations on the data collected and pose questions for further study.

Chapter 4: Interpretations, Findings, and Recommendations

Introduction

In this paper, the author has taken a look at the problem, research question, and why she finds this paper important and relevant. The research question for this paper is: How can cooperative learning be effective in classrooms, specifically math classrooms? This chapter will be dedicated to recapping the ideas that have been presented in the literature review, comparing what has been done with cooperative learning and traditional methods for teaching Foundations Math, revisiting comments made by educators, where the researcher stands with the ideas presented, and other additional comments that should be added before concluding this paper.

Author's Experiences

During the spring of 2003, the researcher student taught in Sauk Rapids, Minnesota, where they had just implemented the Connected Mathematics Project and the Interactive Mathematics Project. Teaching the Connected Mathematics Project and traditional algebra to eighth grade students lead to questions about cooperative learning versus traditional methods.

The Connected Mathematics Project is a standards-based curriculum designed to help students with problem solving and reasoning skills as they derive the different mathematical rules and ideas. There was a great deal of cooperative learning with these classes. In fact, every day included about 20 minutes of cooperative learning. In a typical day students were introduced to an idea, math fact, or concept; they then worked in cooperative groups to derive and discuss the idea presented to them. The students were given adequate time to solve the problem or complete the task, and finally the class as a whole discussed solutions and methods. During the cooperative learning time students worked together to derive the solution or rule, constructively criticized one another, helped others to understand, etc. During the discussion as a class, the groups presented what they did and explained their thinking process. Members of the class could then identify any errors in their thinking and make corrections. The interdependence aspect, introduced in the literature review, of Connected Mathematics presented itself when students were tested on these areas; most assessments were done in groups as well, so the more the group understands the material, the better each student performs on the test. Recall from the literature review that interdependence is when students feel connected in a way that they feel they will not succeed unless they all succeed. From observations of the classes and assessments, the researcher believed that the students in these classes understood the material and enjoyed working with one another on a daily basis.

While attending graduate classes, the researcher encountered many different opinions on cooperative learning. Many peers thought cooperative learning was beneficial because students gained an acceptance for one another. Others thought that cooperative learning improves understanding and creates in-depth understanding of concepts. Some the concerns that were expressed were thoughts of time wasted, whether more advanced students feel it helps them, how the parents of more advanced students feel, whether there would be differentiated assessments for more advanced students, etc. Parents of students in the Sauk Rapids School District also expressed some of the above mentioned concerns. Another concern that Slavin (1990) suggests that educators avoid is what he calls "diffusion of responsibility." This is when there is not enough interdependence among students and some attempt to get their group mates to do all the work.

As a teacher of 9-12th grade students the past three years at Princeton High School, the researcher taught using a primarily traditional format and enjoyed it but felt that she was neglecting the possibility that students could and would learn best from cooperative activities and so the study was appropriate. The following section will discuss the study and interpretations of the data collected, followed by a summary of questions to consider now and in further research.

Thoughts throughout the Study and Interpretation of Findings

After introducing all the roles and spending a lengthy amount of time on the details of cooperative groups with the students, the researcher thought on the first day with group A and B were that students worked well together. Group A is well behaved and so success seemed most certain. Students seemed to understand what was expected of each other and at the end of the sixty-five minutes class the researcher felt confident that they understood the objective.

Students took daily quizzes on the material given when working in cooperative groups. Students took the quiz individually and the scores of all group mates were averaged. Each student got the average score in the grade book. The researcher had some reservations for averaging quiz scores. There was fear that students would be upset that this is done when they could have received a perfect quiz score. The researcher decided to do this averaging anyway to provoke interdependence among students.

Always with decisions like this, there are possibilities that students still choose to not participate, in which case an exception to getting the groups grade would have to take place. It was not necessary in this study but was considered a possibility before hand in the event a student chose not to participate. Students, in study group A participated in learning together and checking solutions with each other. The researcher noticed that students in group A do the problems individually and then compare answers. Group B seemed to need to discuss first and after working each problem. The researcher also occasionally witnessed students copying answers and students just changing answers when their answer does not match their teammates'. Students, at this level, have a tendency not to discuss as much as they should. The researcher saw students believing others, which she can only guess, was due to a lack of confidence in their own work and was the first indication that there could have been more appropriate participants for the study.

Based on the survey results in table 1 and table 3, some possible conclusions are that students did not feel that they received feedback when they gave ideas to the group or did not give feedback when others contributed. The researcher sorted the data into the corresponding groups students were in and it became clear that certain groups had a more negative attitude in general. Some groups just didn't work well together. One cooperative group in Study Group A, did not communicate at all and I seldom heard them speak to each other. This was even after they were encouraged to talk many times.

In Group B, students are much more outspoken and did not need encouragement to work well together the first day. By looking at the survey results it appears as though they did not follow the assigned roles they were given very well. In addition to taking the survey, students were asked to write a sentence or two on how they felt the first day went. Based on group A's responses, the researcher concluded that eight of the students enjoyed working in cooperative groups and four students really felt that working together is unfair for them. Group A was a very independent group who worked well together and worked well individually. Group B was a very social group who, even when asked to complete a task alone had a very difficult time doing so. Group B had two students who did not like working in groups and 12 students that enjoyed it. Complaints from both groups included students not doing there assigned task, student talking about non topic issues, and students relying on others to do the work.

These comments by students were surprising. The researcher expected to get more negative responses. This is an interesting and thought provoking outcome. One question that arose from reading comments from the students was: How satisfying and thought provoking is the average traditionally formatted lesson? It would be interesting to see results of a survey given to a traditional class on lesson satisfaction. The researcher organized these sheets into corresponding groups and found that two of the five groups in study group A had all members that were happy and very satisfied with working with others. The other three groups had a minimum of one unhappy and unsatisfied member. Study group B had mostly happy group members except for one member in two groups who really preferred working alone.

On the fourth day of both study groups, the researcher asked students to complete the same survey and make comments on cooperative learning. Students in study group A appeared to be working better together. This is good to see and may indicate an acceptance of one another and their ideas, but with fewer expressing their ideas it causes some concern. Study group B seemed to also be enjoying it more and figuring out how to work productively with each other. In group B, the students argued about correct answers. The researcher was glad to see the disagreement but was very unimpressed with the methods students used to resolve the conflict. Students spent more time going back and fourth with the "you're wrong" comment then trying to persuade the others with mathematically structured proofs. The teacher spent considerable time with group B on conflict control, classroom management and behavior management. Even though this was the case, the researcher pondered whether students were gaining an acceptance of one another as suggested in the literature review and cited by Battista(1999) and Kegan(1994).

Despite the issues the researcher was having with group B, she continued to do cooperative work past the sections group A had done cooperatively. One question that arose for the researcher was: If students have no tolerance for one another, is it beneficial to then, perhaps not just mathematically, but for social reasons to continue cooperative group activities? Will students acquire an acceptance for one another if they are asked to work with them regularly? This encouraged the researcher to continue.

Study group B spent the last two chapters of the trimester doing the cooperative activities that went along with the lessons. This group of students became less interested in mathematics and more interested in who was going to be in their group that week. They complained when they saw who they were placed with. Groups were formed in a variety of ways, as discussed in Owens (1995). Some times they were grouped by ability, random grouping, and by opposing ability. Students were not told which way they were grouped, however, students today can easily identify struggling students or the student that is not challenged enough. The class dynamics were very difficult to structure lessons around. Most students in group B had issues outside of school that made doing well in school a low priority, which again may be an indication of incorrect study participants.

The grades these students exhibited were astonishing to the researcher and invoked even more questions. Based on table 5 in Chapter III, where we look at the common assessments between the two classes, we can see that until the end of Chapter six students in both groups had approximately the same averaged in most of the assessments. After Chapter six, students in group A went back to tradition structured lessons and group B stayed with cooperative activities. Based on this table, we can see that group B has an average that is much lower than group A. Which provokes the question: Is this because group A was on a well structured, traditional format with very clear expectations and group B was on a cooperative learning, less pressure, and an open for discussion and argument structure?

A final thought and a conclusion from the researcher is that well structured traditional formats and cooperative learning activities are appropriate for certain lessons. Surely one cannot and will not be appropriate all the time. Based on the teachers experience with the topic he or she should come up with the most appropriate format for the group of students he or she has at that time. These decisions should be based on class dynamics, ability and type of lesson.

Possible Failures in this Study

There are other things to consider before determining if cooperative learning is more or less effective then traditional methods. The two study groups were given at different times of the year and at different times of the day. Study group A was first period everyday and was second trimester of the year. They met from 8:10 to 9:15 am from December 3 to March 7. Students in this trimester benefitted from the first hour class when students are calmer and are more open to acquiring new information. They are also in a trimester with a two week winter break where they can rest from their studies and allow information to sink in a bit. Students in group B could have been hindered by the time of year in which they took this class. Group B took place from 12:35 to 1:40 pm from March 11 to June 5. Students in this group took the class after lunch. There is the possibility that students are less engaged at this time of the day and focus less resulting in lower averages. Students in this group also have class form March to June, when school is let out for the summer, which could contribute to lower averages as well. There is a possibility that group A would have the same low averages had they taken the class after lunch and at the end of the school year. Do students lack focus and interest after lunch? Do students lack focus and interest at the end of the school year? These are some questions that could be researched further in order to really examine the results found in this study.

Another possible failure to this study is the bias of the instructor to cooperative learning coming out in such a way that affects the students' ability to be successful. Because the researcher was looking at the results there is the possibility that she allowed students to perform lower unknowingly.

Other considerations in the study are classroom dynamics. Do students with certain types of disorders affect the overall interest and focus of the other students in the class? An example to consider and do further research would be if a student, like in study group B, has accommodations for defiant behavior. This takes attention away from the class to address issues and conduct conflict resolutions. Furthermore, this study was conducted with two classes that had a large number of special needs students in them. Is it acceptable to make generalizations based on classes that have roughly a sixty percent special needs and at risk rate? Would the outcomes be the same if the classes had no special needs or at risk students?

This study was only conducted with Foundations students which could be a possible error in judgment by the researcher. Cooperative learning could be more effective with a more mature student base. The researcher feels, at this point, that a Geometry curriculum would be very productive with many cooperative learning activities. Teachers must construct or search to find the right curriculum for the groups, as suggested by Davison (1990). A Geometry course would lends itself well to discussion and group discovery type of activities. This afterthought provokes possible further research in the subject of Geometry.

Recommendations for Further Studies

The study conducted by the researcher seemed very concrete at the beginning with picking two of the same class and ability and comparing results when implementing cooperative learning activities in one and applying a traditional format in the other. It is very evident that further research needs to be done in order to come to a conclusion about the effects of implementing cooperative learning in the classroom. Some questions that arose due to the study and should be researched further are:

1. How satisfying and thought provoking are traditionally structured lessons?

2. Do students lack focus and interest at the end of the school day? After lunch?
3. Do students lack focus and interest at the end of the school year?
4. Do students with certain types of disorders affect the overall interest and focus of the other students in the class?
5. Is the curriculum conducive to cooperative learning or should a more appropriate curriculum be implemented?

These questions and further study groups with parameters set to prevent failures to the study are needed to answer the central question of this paper. Resources for future reading on cooperative learning are given in Appendix C.

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Appendix A

Addison-Wesley: Foundations of Algebra and Geometry Sections

6-2 PART A

Angles and Angle Relationsh

← CONNECT → *Angle sizes determine the shape and size of physical objects. You will measure angles and explore angle relationships to prepare for work with similar figures.*

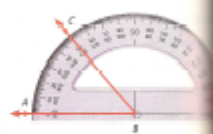
To find ratios in geometric figures, you need to measure the lengths of their sides and the sizes of their angles.

You have used a ruler to measure distances along a line. Units for measuring distances include inches, centimeters, feet, miles, and kilometers. A **protractor** is a tool for measuring angles. Angles are measured in degrees.

The protractor shows that the measure of $\angle ABC$ is 50°.

We write this in a special way:

$m\angle ABC = 50°$.



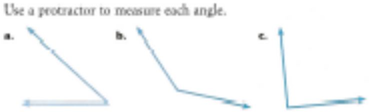
CONSIDER

1. **How would you describe an angle with the measure of 180°?**
2. **How do you know that the measure of $\angle ABC$ above is 50° and not 130°? How can you tell whether to use the top markings or the bottom markings on the protractor for the measure of an angle?**
3. **Suppose each side of the 50° angle above, $\overline{BA}$ and $\overline{BC}$, is increased from $1\frac{1}{2}$ inches to 3 inches long. How does this change the measure of the angle?**

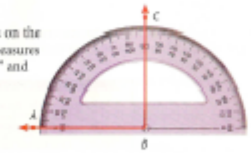
TRY IT

Use a protractor to measure each angle.

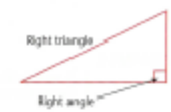
a. b. c.



Notice the 90-degree marks on the protractor. An angle that measures 90° makes a "square corner" and is called a **right angle.**



In drawings, right angles are usually marked with a small square.



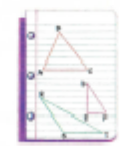
A triangle that has a right angle is called a **right triangle.**

EXPLORE: SUM IT UP

MATERIALS

Ruler, Protractor

Use a ruler to draw three different triangles on your paper. Your triangles should be similar in shape to these three triangles.



Make each one large enough so that you can measure the angles with a protractor easily. For each triangle, measure and record the sizes of the three angles.

1. What is the sum of the measures of the angles in $\triangle ABC$? in $\triangle DEF$? in $\triangle RST$?
2. What conclusion can you draw about the sum of the measures of the angles of a triangle?

Repeat the process using three different quadrilaterals, shaped similar to these.

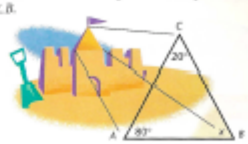
3. What is the sum of the measures of the four angles in $ABCD$? in $GHJK$? in $WXYZ$?
4. What conclusion can you draw about the sum of the measures of the angles of a quadrilateral?
5. What relationship can you find that would relate a quadrilateral to two triangles?

Your measurements in the Explore may have confirmed the following.

> The sum of the measures of the angles of a triangle is 180°.
> The sum of the measures of the angles of a quadrilateral is 360°.

EXAMPLE

A sand castle has a tower with a triangular roof shaped like $\triangle ABC$. Find the measure of $\angle B$.



Using the angle-sum relationship above, we can write an equation.

$$m\angle A + m\angle B + m\angle C = 180$$

$80 + x + 20 = 180$ — Substitute values.

$x + 100 = 180$ — Simplify.

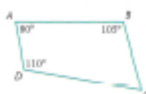
$x = 80$ — Subtract 100 from each side.

The measure of $\angle B$ is 80°.

In the Example, notice that both $\angle A$ and $\angle B$ measure 80°. Recall that angles with the same measure are **congruent angles.** In symbols, we write $\angle A \cong \angle B$.

TRY IT

d. Find the measure of $\angle C$.



REFLECT

1. How do you position a protractor to measure an angle? How do you read the angle measure?

2. Suppose the three angles of a triangle are congruent. What is the measure of each angle?

3. One angle of a quadrilateral measures 90°. Do the other three angles also each measure 90°? Explain.

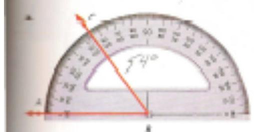
4. What could the term *congruent* mean for sides of a triangle? for the whole triangle?

Exercises

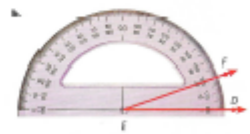
CORE

1. **Getting Started** Find the measure of each angle.

a.



b.



2. The sum of the measures of two angles of a triangle is 165°. What is the measure of the third angle?

Use a protractor to find the measure of each angle.

3.

4.

5. One angle of a triangle measures 90°. What is the sum of the measures of the other two angles?

(a) 90° (b) 180° (c) 270° (d) not here

6-2 PART B Similar Figures

CONNECT → *You have seen that the sum of the measures of the angles in a triangle is 180°, and in a quadrilateral it is 360°. You will use angle measures and proportional sides to develop the idea of similar figures.*

Blueprints, maps, and computer chips are all examples of the use of similar figures. The blueprint shows a reduced version of a building; a map is a reduced version of roads and streets; computer chips are photographically reduced versions of large-scale drawings.

These three photographs of the castle are **similar.** One is the original photo, one is an enlargement, and the third is a reduction.

SIMILAR

\'sim-ə-lər adj.

1: having characteristics in common; strictly comparable

2: alike in substance or essentials

CONSIDER

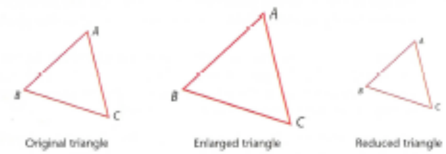
1. What features of these three photos are exactly the same?
2. What features are different?

You will need to explore relationships in similar figures so that you can understand the mathematical use of the term *similar* better.

EXPLORE: IT'S OK TO COPY!

MATERIALS: Centimeter ... Protractor ... Copy machine

1. Make a large drawing of a triangle. Use a copy machine that has a Reduce/Enlarge control to make an enlargement and a reduction.



2. Use a ruler and a protractor to measure all the sides and angles of the figures on your original drawing and the two copies. Record the angle measurements on the figures. Record the lengths of the sides in the table below.

Triangle Sides	Original Triangle	Enlarged Triangle	Reduced Triangle
$\overline{AB}$			
$\overline{BC}$			
$\overline{CA}$			

3. The three angles at *A* are called **corresponding angles.** The three angles at *B* and the three at *C* are also corresponding angles. Compare all the measures of corresponding angles. What do you find?

4. Find these ratios of **corresponding sides.**

$\frac{\text{Original length } AB}{\text{Reduced length } AB}$ $\frac{\text{Original length } BC}{\text{Reduced length } BC}$ $\frac{\text{Original length } CA}{\text{Reduced length } CA}$

Find these ratios of corresponding sides using the enlarged lengths.

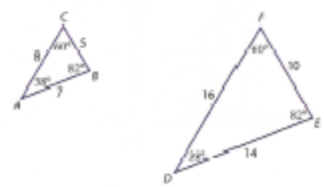
$\frac{\text{Original length } AB}{\text{Enlarged length } AB}$ $\frac{\text{Original length } BC}{\text{Enlarged length } BC}$ $\frac{\text{Original length } CA}{\text{Enlarged length } CA}$

5. Write a statement about the ratios of **corresponding sides** of your triangles.

If two polygons can be matched up so that corresponding angles are congruent and corresponding sides have the same ratio, we call them **similar polygons.**

EXAMPLE

1. Show that the two triangles are similar.



Begin by checking the corresponding angles.

$\angle A$ and $\angle D$ both measure 38°.

$\angle B$ and $\angle E$ both measure 82°.

$\angle C$ and $\angle F$ both measure 60°.

So the corresponding angles are congruent.

Next compare the ratios of corresponding sides.

$\frac{AB}{DE} = \frac{7}{14}$, or 0.5

$\frac{BC}{EF} = \frac{5}{10}$, or 0.5

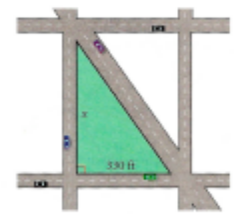
$\frac{AC}{DF} = \frac{8}{16}$, or 0.5

Corresponding sides have the same ratio. So triangle ABC is similar to triangle DEF. We write $\triangle ABC \sim \triangle DEF$.

If two triangles are similar, you can use proportions to find the lengths of unknown sides.

EXAMPLE

2. Sandy Boulevard is a diagonal street that cuts through a rectangular grid of streets. This creates a few triangular lots. How can you find the missing length by using these similar triangles?



$\frac{2.5}{330} = \frac{3.5}{x}$ Use corresponding sides to build a proportion.

$2.5 \cdot x = 330(3.5)$ Cross multiply.

$2.5x = 1155$

$x = 462$

The missing length is 462 feet.

You can experiment on enlargements and reductions of figures that have 4, 5, or 6 sides. You will find that all corresponding angles are congruent and all corresponding sides have equal ratios.

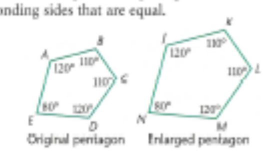
EXAMPLES

3. Pentagon $JKLMN$ is an enlargement of pentagon $ABCDE$. Find three ratios of corresponding sides that are equal.

$\frac{AB}{JK} = \frac{CD}{LM}$

$\frac{BC}{KL} = \frac{AE}{JN}$

$\frac{CD}{LM} = \frac{ED}{NM}$



4. The photo on the right is a reduction of the one on the left. What is the width of the reduced photograph?

The rectangular photos are similar.

$$\frac{\text{Width of original}}{\text{Width of reduction}} = \frac{\text{Height of original}}{\text{Height of reduction}}$$

$$\frac{4}{3.2} = \frac{3}{h}$$

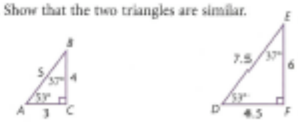
$$4h = 9.6 \quad \text{Cross multiply.}$$

$$h = 2.4$$

The reduced photograph is 2.4 inches high.

TRY IT

a. Show that the two triangles are similar.



b. The two figures are similar. Find the missing length.

REFLECT

1. How can you tell if two triangles are similar?
2. Are all squares similar? Are all rectangles similar?
3. If $\triangle ABC$ is similar to both $\triangle DEF$ and to $\triangle PQR$, will $\triangle DEF$ and $\triangle PQR$ be similar? Explain.

6-2 PART C Blow It Up and Scale It Down

← CONNECT → ***You've learned how to work with ratios and proportions. You also know some properties of similar polygons. Now you can combine these two ideas to make scale models and scale drawings.***

Builders of sand sculptures might make miniature castles, or they might make giant monsters. For realism, these sculptures should be similar to the original objects. The miniature castle is built on a small scale, and the monster is built on a large one.

Actual [illegible]

Sand model

These figures are similar. Notice how corresponding sides compare. The **scale factor,** or **scale,** is the ratio of corresponding lengths.

$$\frac{\text{Enlarged length}}{\text{Original length}} = \frac{15 \text{ ft}}{5 \text{ in.}} = \frac{180 \text{ in.}}{5 \text{ in.}} = \frac{36}{1}$$

The enlargement has a scale factor of 36 to 1, or 36:1.

If the denominator of the scale factor is 1, we may omit writing it. So if an enlargement has a scale factor of 36:1, we say that its scale is 36. That is, its linear measurements are 36 times those of the original.

EXPLORE: MOVIE SCALING

Honey, I Shrunk the Kids was a popular movie that "scaled down" the problem of raising children.

Imagine yourself as one of the young stars in the movie. You need to shrink yourself.

1. What scale factor will you use? Why?
2. How can you use your scale factor, along with your own measurements, to decide the measurements of your shrunken self?
3. You need to give measurement information to the costume department. Copy the chart. Complete it by taking measurements and doing the necessary calculations.

	"Real Self" Measurements	"Shrunken Self" Measurements
Height		
Arm length		
Thumb length		
Wrist circumference		
Head circumference		
Knee-to-ankle length		

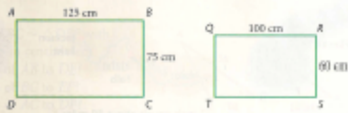
4. What scale factor would you use if you wanted to be small enough to hide in a cookie jar?
5. What scale factor would you use if you wanted to be small enough to take a shower in the kitchen sink, but big enough not to fall down the drain?
6. Discuss the effect of different scale factors on your "movie." Write some conclusions to share with your class.

CONSIDER

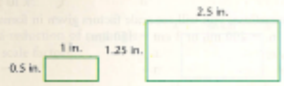
1. **A model airplane has a scale factor of 1:50. What does this mean?**
2. **Can an object have more than one scale factor?**

TRY IT

a. Rectangle $QRST$ is a reduction of rectangle $ABCD$. What is the scale factor?



b. What is the scale factor for the small rectangle below?



WHAT DO YOU THINK?

A company in South Dakota wants to design a model of Mt. Rushmore to sell as a souvenir. They know that Lincoln's head measures 66 feet tall. If the scale factor is 1:120, how tall will the model of Lincoln's head be?

Ben thinks ...

I can multiply by the scale factor.

The measurements of the model are $\frac{1}{120}$ the measurements of the original.

$\frac{1}{120} \cdot 66 \approx 0.55$

The model of Lincoln's head will be 0.55 feet high.

Elena thinks ...

I can set up a proportion.

$$\frac{\text{Model height (ft)}}{\text{Real height (ft)}} = \frac{1}{120}$$

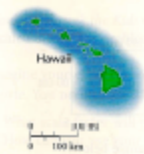
$$\frac{m}{66} = \frac{1}{120}$$

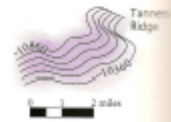
$$120m = 66 \quad \text{Cross multiply.}$$

$$m = \frac{66}{120}, \text{ or } 0.55 \quad \text{Divide both sides by 120.}$$

The model of Lincoln's head will be 0.55 feet high.

On a map, a *legend* may describe the scale. Scale may be described with pictures, or words, or both.





Sometimes you will see scale factors given in forms such as 1 in. = 3 ft, 1 in. = 200 mi, or 3 cm = 150 km.

EXAMPLE

The scale on a map of Ohio says 1 inch equals 25 miles. The distance on the map from Toledo to Cincinnati is approximately 9 inches. About how many miles apart are these two cities?

Set up a proportion using two ratios in the form:

$$\frac{\text{Distance in inches}}{\text{Distance in miles}}: \frac{1 \text{ inch}}{25 \text{ miles}} = \frac{9 \text{ inches}}{x \text{ miles}}.$$

$1 \cdot x = 25 \cdot 9$

$x = 225$

It is about 225 miles from Toledo to Cincinnati.

REFLECT

1. What does it mean if a blueprint of a house is drawn on a scale of $\frac{1}{2}$ in. = 1 ft? Would this be a good scale factor for an architect to use? Explain.
2. Suppose you build a model with a scale factor of $\frac{2}{3}$. Is the model larger or smaller than the original?
3. What does a scale factor of 0.4 mean? What does a scale factor of 1 mean?
4. A map legend says 1 inch = 10 miles. How would you use this information?

6-2 PART D The Pythagorean Theorem

← CONNECT → ***You have seen that similar triangles are very useful in practical ways. Now you will see that right triangles that are similar have some special properties that make them valuable in finding unmeasurable dimensions.***

Right triangles have special names for their sides.

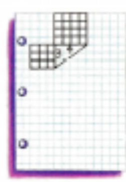
The two sides of the right angle are called **legs**.

The long side opposite the right angle is called the **hypotenuse**.

EXPLORE: FAIR AND SQUARE

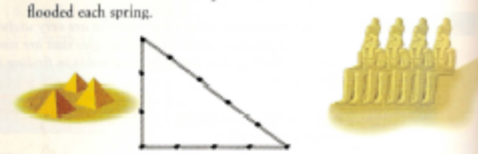
MATERIALS

Graph paper, Scissors, Ruler
Several 3-by-3, 4-by-4, 5-by-5, and 12-by-12 graph paper squares



1. On graph paper draw a right triangle with legs of 3 units and 4 units. Leave room to draw a square on each leg as shown.
2. Explain why it is not easy to tell the length of the hypotenuse by counting squares on the graph paper.
3. Cut a 3-by-3 square and a 4-by-4 square apart into individual squares.
4. Can you arrange these squares into a large square along the third side of the triangle? If so, what length do you find for the hypotenuse?
5. Repeat for triangles with legs of 5 and 12.
6. What relationship can you find among the squares that you have drawn for each triangle?
7. What are the values of 3^2, 4^2, and 5^2? How are they related? What are the values of 5^2, 12^2, and 13^2? How are they related?

You have been working with one of the most famous discoveries in mathematics. Ancient Egyptians used ropes with knots spaced at three, four, and five units to make square corners on their fields after the Nile flooded each spring.

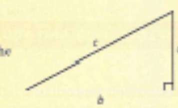


This discovery is called the Pythagorean Theorem after the Greek mathematician, Pythagoras.

THE PYTHAGOREAN THEOREM

If a and b are lengths of the legs of a right triangle, and c is the length of the hypotenuse, then

$a^2 + b^2 = c^2$.



This relationship is true for all right triangles, and only for right triangles.

You can use this relationship in two ways:

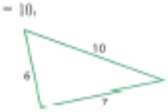
- to see if a triangle is a right triangle, or
- to find the length of a side of a right triangle, if the other two lengths are known.

EXAMPLES

1. A triangle has sides of 6, 7, and 10. Is it a right triangle?

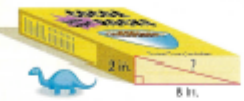
Let $a = 6$, $b = 7$, and the hypotenuse $c = 10$.

$$a^2 + b^2 \stackrel{?}{=} c^2$$
$$6^2 + 7^2 \stackrel{?}{=} 10^2$$
$$36 + 49 \stackrel{?}{=} 100$$
$$85 \neq 100$$



The triangle is not a right triangle.

2. The bottom of a cereal box is 2 in. by 8 in. A toy manufacturer wants to be sure that a toy will not end up stuck in the bottom of the box. How long is the diagonal of the box?



We want to find the length of the hypotenuse. Let the legs be a and b, so $a = 2$ and $b = 8$.

$a^2 + b^2 = c^2$ — The triangle is a right triangle, so use the Pythagorean Theorem.

$2^2 + 8^2 = c^2$

$4 + 64 = c^2$

$c = \sqrt{68}$

$c = 8.25$ — Use the square root key on a calculator to find the square root of 68.

The diagonal of the box is about 8.25 inches long.

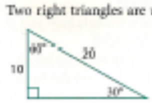
CONSIDER

1. How can the toy manufacturer use the information above?

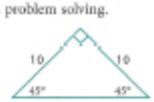
TRY IT

a. Find the length of the hypotenuse of this triangle.

Two right triangles are used often in problem solving.



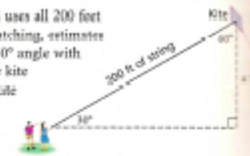
In any 30°-60°-90° triangle, the short leg is half the length of the hypotenuse.



In any 45°-45°-90° triangle, the two legs are the same length.

EXAMPLE

3. Felice is flying a kite and uses all 200 feet of string. Neil, who is watching, estimates that the string makes a 30° angle with Felice's hand holding the kite string. How high is the kite above her hand?



The kite diagram makes a 30°-60°-90° triangle.

$x = \frac{1}{2}(200)$ — The short side is half the hypotenuse.

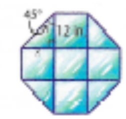
$x = 100$

The kite is 100 feet above Felice's hand.

In the example above, it would be difficult to measure the height of the kite directly. Using what we know about triangles and their relationships lets us find these distances by mathematics. This is often called *indirect measurement*.

TRY IT

b. A window is made up of panes, as shown. What is the length of x?



REFLECT

1. How can triangle information be used to find distances that you cannot measure directly?
2. A phrase such as "directly overhead" implies that there is a right angle in a situation. What other words or situations imply right angles are present?
3. How do the side lengths of a triangle help you determine if the triangle contains a right angle?
4. In a right triangle, how can you find the length of a leg if you know the lengths of the other leg and the hypotenuse?
5. If a right triangle contains a 30° angle, what else do you know about the triangle?
6. If a right triangle contains a 45° angle, what else do you know about the triangle?

Appendix B

Self-Evaluation Survey

GROUP LEARNING SELF-EVALUATION

Name ______________________ Date ______________________

Group ______________________

Read each statement and rate your group 4 if you agree with the statement, 3 if you somewhat agree, 2 if you somewhat disagree, or 1 if you disagree. Use NA, not applicable, if the statement does not apply in this situation. Circle one response for each description of your group.

Members of the group...	agree	somewhat agree	somewhat disagree	disagree	not applicable
performed their assigned roles.	4	3	2	1	NA
understood the purpose of the Explore.	4	3	2	1	NA
understood the solution to the Explore.	4	3	2	1	NA
were able to answer the Consider and Try It.	4	3	2	1	NA
listened to each others' ideas.	4	3	2	1	NA
gave feedback to those who contributed ideas.	4	3	2	1	NA
stayed on task.	4	3	2	1	NA
assisted in preparing the work that was collected.	4	3	2	1	NA
had their assignment from the previous day.	4	3	2	1	NA
expressed their ideas to the group.	4	3	2	1	NA
were willing to compromise when needed.	4	3	2	1	NA
actively participated in the group.	4	3	2	1	NA
______________________	4	3	2	1	NA
______________________	4	3	2	1	NA

Appendix C

Resources for Future Reading on Cooperative Learning

Resources for Further Reading on Cooperative Learning

Miss Regnier December 2004

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