

Student Centered Mathematics in an Isolated Skill Environment-
Constructivist Methods in Mathematics

by

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STATEMENT BY AUTHOR

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THE IMPORTANCE OF CONSTRUCTIVISM IN A MATHEMATICS CLASSROOM

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Chapter 1: Introduction

Theorists and researchers have been debating for years what is the best way to teach children. As the pendulum has swung from very teacher directed instruction to very student directed instruction our school has aligned its philosophy to a very student centered, constructivist approach. Now, with new information coming out about the brain (also from Vygotsky's work) we can see that communication and delivery of the subject material may be just as important as the content and the program. The National Council of Teachers of Mathematics states that all students are capable of learning mathematics and that it is the teacher's responsibility to provide rich experiences where students are able to construct ideas about concepts and can discuss these constructs in an effective manner.

This year, our school district decided to adopt the Common Core State Standards in anticipation of our state department of education adopting this curriculum. Wyoming did decide to adopt the curriculum. As of April 2013, all but five states have adopted the CCSS. This has impacted mathematics instruction in our district and has ultimately driven the individuals in decision-making roles for Natrona County School District to find and implement one mathematics program that focuses on the Common Core State Standards. A Mathematics Adoption Committee was created in October of 2012. The committee's goal was to recommend the most aligned mathematics resources Kindergarten through twelfth grade to the CCSS. This committee was composed of educators in the district who applied for the position. Officials from Natrona County School District reviewed the applications and chose educators. Committee members went through professional development activities designed to educate on the CCSS.

Three publishers submitted programs that are aligned to the curriculum. These publishers include Houghton Mifflin, McGraw Hill and Pearson. Ultimately, the committee chose McGraw

Hill's My Math. This program focuses more on isolated skills rather than placing students in problem solving situations. This program touts itself as a colorful and fun way to teach mathematics as described by the curriculum sales department. It also is very verbal about its ability to help with data-driven instruction, a catch phrase our district has adopted over the last three years.

Natrona County School District is considered a "school of choice" district. Parents are invited to choose the type of education that best fits their child. We have varied schools and varied programs at each school. 'Back to Basic' education is offered at one school while others offer very student driven, constructivist instruction. Many schools in our district currently use Bridges in Mathematics, others use Everyday Mathematics and the "Back to Basic" school uses Saxon. Because of our adoption of the CCSS, we will be adopting one program that is specifically written to these standards. The question is: Will this program deliver mathematics instruction in a way that will make mathematics an approachable, exciting subject worth discovering, or will it simply teach children the steps involved in an algorithm situation? If teachers teach this program with fidelity, will the students receive enough conceptual knowledge prior to moving into the procedural knowledge that is heavily relied upon in this program?

Prior to this adoption, our school focused on improving our mathematics scores. In 2003 our school's (school A) NWEA Growth Assessment scores were critically low. Our school was placed on an improvement watch list. School A decided to look carefully at programs that were aimed at student led instruction, which led to the adoption of Investigations. Investigations is a constructivist program developed through grant support from the National Science Foundation and focused on ideas recommended by the National Council of Teachers of Mathematics. It includes an investigative model of learning where the students are given problems and led in

investigations to discover mathematical reasoning and solutions to problems.

After two years of implementation, teachers and administrators decided Investigations was too difficult to implement in the classroom. Teachers struggled with the format of the teaching guide and the lack of professional development received. School A found a compromise with another program being used in the district and adopted Bridges in Mathematics. This program has a primarily constructivist focus. Students construct meaning of mathematics by creating and sharing strategies by which they solve their problems. When combined with the Number Corner portion of the program it is also very problem based. Questions are posed to the children and they are responsible for finding a solution and sharing the strategy. The educator also introduces strategies but the students become ultimately responsible for the strategy that works for the student. They also must be able to explain the process used.

Prior to implementation of Bridges in Mathematics, the school was using a traditional textbook from the Scott Foresman collection. School A looked at the students' performance on NWEA testing and decided that a change was necessary.

This research project explores the extent to which a teacher's delivery methods and instructional philosophy can improve children's abilities in the area of mathematics. The research question arises from the need to analyze how delivery and open exploration can improve a child's ability to become a more competent problem solver, and mathematician.

Statement of the Problem

The purpose of this paper is to study the way delivery of curriculum in the classroom can enhance or hinder student learning in the area of mathematics. This will be done by reviewing literature in four primary areas:

- a) current and historical literature on constructivist methods in the classroom
- b) how this philosophy can be implemented in our current environment of reliance on test scores and the way in which children's achievement is being measured
- c) can a child receive a meaningful education in mathematics through a traditional educational environment? The researcher defines meaningful education in producing students who are able to apply mathematical skills in varying situations in an independent and collaborative manner.
- d) can a teacher teach in a constructivist and problem based manner when a district has a required program for teaching toward a traditional delivery method?

I will examine scores from School A related to NWEA testing prior to and after our school adopting a more constructivist model of teaching. This will help the researcher answer the question of the delivery method.

Limitations

This paper focuses on how constructivist methods affect the classroom and the students. Although the author will look at the differences between a traditional classroom and a constructivist, problem-based classroom, delivery in a traditional classroom will not be an area of focus. Instead, I will focus on effectiveness of constructivist methods.

Another limitation of the paper is that only one school's test scores will be considered when determining the success of a constructivist program. The scores are from a test primarily geared toward a student's abstract application through questions given in traditional algorithm format. The NWEA Growth Assessment is primarily geared towards a student's ability to

perform computation and the problems on the test are not frequently given in a problem-solving manner. This test does not directly measure a student's problem solving abilities, which is an important aspect of constructivist teaching.

Definition of Terms

Constructivism is defined as a teaching method where the student is constructing his or her own knowledge from exposure to specific educational concepts.

Cooperative learning is a method where instruction is organized into academic and social experiences. Students work together to solve problems.

Traditional teaching methods refer to students being given the knowledge they need based on a series of steps. Students learn a new skill, as given by the teacher, and practice the skill in isolation until mastery is observed. This is also referred to as the procedural-formalist paradigm (PFP). This theory states that learning mathematics requires the educator to break down mathematics into a smaller group of isolated skills, facts and procedures and the realization that this knowledge “exists apart from the human experience” (Berry III, Ellis 2005)

Chapter 2: Literature Review

To begin, one needs to understand the differences between a traditional and constructivist classroom. For years, teachers, administrators and legislators have debated the best way in which to teach mathematics. Traditional methodologies had been the status quo in mathematics classrooms. The teacher would introduce a skill and the students would practice that skill until a quiz or test showed mastery. This resulted in only some students being competent in the specific skill and very few able to take these abstract concepts and apply them to problem solving situations. Most students were unable to take this knowledge and transfer it to different situations. Mathematics was taught as a subject in isolation. The need to make mathematics more transferable and meaningful was a driving force to help curriculum developers. (von Glaserfeld, 1991)

In the early 19th century educators and psychologists began seriously looking at the “art” of teaching mathematics. In mathematics there are two types of changes that have been taking place in mathematics education revisions and reform (Ellis & Berry, 2005). They classify revisions as “a renewal effort that captures educators’ attention for a short period of time but fails to address critical issues that are at the root of the students’ difficulties with mathematics.” (Ellis & Berry, 2005, p.7) Reform, on the other hand, refers to questioning key beliefs about how best to teach mathematics, looking at the nature of mathematics, and defining the requirements for success for all students in learning mathematics.

As educators began looking at how to serve the needs of students in the early 1900’s Edward L Thorndike began working with educational psychologists to determine how to educate the increased number of students entering public schools. It was during this work that Thorndike composed his theory of Stimulus-Response Theory (Thorndike, 1923), which stated that students

would learn mathematics in an environment of drill and practice. Thorndike and his colleagues concluded that mathematics should be taught in a carefully sequenced manner in which an educator shows a child the specific steps to complete a problem followed by extensive practice on the steps and skills. This theory led education in mathematics during this time.

In the 1920's a group calling itself the Progressive Education Association (PEA) began looking at John Dewey's (1899) work. They noted the importance of society and a child's innate instinct toward learning. This thinking led educators to look at the need to include a child's experiences and interests in their education. The PEA declared many principles important to education including the need to allow children to develop naturally, work motivated by interest, and the teacher's role as one of guide, not taskmaster. These principles stood against the work of Thorndike and many educators regarded these ideas as far too radical. The PEA had little influence on education during its time.

Later, the social efficiency movement took some of the work done by the PEA to compose new ideas about education. The major idea that was adopted was the thought on the learner as an individual and this notion was twisted into a new idea that only those who needed it should learn advanced mathematics. In the social efficiency model researchers questioned the need for formal mathematics education delivered to the masses. Social efficiency progressives turned toward the science of standardized testing to prove their theory that some children were more capable than others in mathematics, therefore rendering the need to teach every child unnecessary. The belief was that students should only learn what they would need to know for their supposed future. Advanced mathematics courses were offered only to those students who had a gift in the subject area as determined by traditional testing and who might need the coursework in their futures. Incredible numbers of students stopped taking mathematics classes

in high school, a time in which the population of public school students boomed. In 1905, fifty-seven percent of students were taking algebra classes. In the late 1940s and early 1950 only twenty-five percent of high school student were enrolled in algebra.

In 1950 Congress created the National Science Foundation. Creation of this foundation was to drive science education due to the need to further national security. Administrators observed the advancements being made in other countries, which drove the need to increase United States students' abilities in the areas of science and mathematics. During this time extensive research was done. In the beginning, multitudes of different tactics were adopted to help students learn mathematics. Theories of early pioneers were being implemented in textbooks and classrooms. Textbooks by Henry Van Engen and Maurice Hartung were being taught. Manipulatives in the classroom, an earlier idea by Catherine Stern were being used, algebra was being taught in inner-city junior highs, and other non-traditional methods were being implemented to help students. Although these activities were unsuccessful in achieving widespread influence, they proved important to upcoming educators who saw the need for reform instead of simple revision.

In 1955 the College Entrance Examination Board (CEEB) created and administered Advanced Placement (AP) testing. Calculus was the first assessment given to AP students. Results lead the CEEB to conclude that significant changes needed to be made in mathematical education in order to prepare high school students for collegiate mathematics. Unfortunately, Russia's launch of Sputnik in 1957 led the United States Government to become impatient with the reform movement of the time. Instead some funding was pulled from the NSF and was used to create the School Mathematics Study Group (SMSG). The SMSG immediately looked at the data from the CEEB and concluded that modern mathematics needed to be taught to all students.

This movement was called New Math. Textbooks were composed and sent to classrooms nationwide. In the haste to create efficient mathematicians, pedagogical innovations from earlier theorists and educators were largely forgotten and New Math with content including new set theory and Boolean algebra, was delivered to students throughout the United States. Students went from little to no instruction in these concepts to advanced mathematics content.

After this movement ultimately failed, a new movement was created to repair the shortcomings of New Math. This movement, happening in the 1970's was considered a "back to basic" approach. This movement went back to Thorndike's philosophy of repeated practice and breaking down problems into isolated skills. "This movement called for decontextualized and compartmentalized skills-oriented mathematic instruction" (Ellis & Berry, 2005, p. 10). At the same time the United States was also giving new assessments to its students; minimum competency testing. New textbooks were composed and sent to students everywhere.

According to Ellis and Berry, "the behaviorist science of Thorndike's psychological models, the vocational focus of the social efficiency progressives, the curricular elitism of the New Math program, and the shallow content of the back-to-basics movement have all been referred to as efforts to reform mathematics education but, for the most part, have resulted in superficial revisions to standard practices and outcomes" (2005, p. 10).

Finally, in the late 1970's, early 1980's true shifts in mathematical education began to take place. In 1983 the National Commission on Excellence in Education released findings from a two year investigation in the document *A Nation at Risk; the Imperative for Educational Reform*. Researchers compared schools both nationally and internationally. Their findings were condensed into five specific areas and the commission gave 38 specific recommendations. This report of our deficiencies in the area of education combined with the nation's renewed interest in

the teaching of mathematics and the advancements being made in technology, educators were realizing the importance of teaching students to be true learners in mathematics. Although many had known that reform was a necessity in early generations, Americans were finally ready to delve into looking at mathematics education in innovative ways. Mathematics educators began looking at Piaget's and Bruner's theories more closely and ways to implement these philosophies into classrooms across the nation. Companies such as Math Learning Center were created out of a movement from the National Science Foundation in an effort to improve mathematics instruction. The goal of these types of organizations was to improve conceptual understanding before moving students into the abstract world of the algorithm and the procedural "short-cuts" that mathematicians use. The use of manipulatives, cooperative learning environments, discussion and sharing of personal strategies and problem solving situations were emphasized.

Ellis and Berry (2005) coined the traditional method of teaching skills to mastery as the procedural-formalist paradigm (PFP). This style of teaching relied heavily on early work done by Skinner (1953), Bloom (1956) and Gagne (1965) and fall under the behaviorist theory of learning. In the procedural formalist method these theorists concur that instruction should be based on the thought that "proficient skills will quantify to produce the whole, or more encompassing concept" (Fosnot, 1996).

On the opposite end of the spectrum is the cognitive-cultural paradigm. This paradigm concludes that "mathematics (is a) set of logically organized and interconnected concepts that come out of human experience, thoughts, and interaction-and that are, therefore, accessible to all students if learned in a cognitively connected and culturally relevant way" (Ellis & Berry, 2005, p.12). This paradigm focuses on the human element in learning and the connection to the cognitive and sociological theories and studies of the experts in the field of cognitive science

such as Jean Piaget, 's work on equilibration of cognitive structures (1977) Lev Vygotsky's zone of proximal development and use of dialogue in learning (1962/1986), Jerome Bruner's finding of the importance of scaffolding in learning (1978), and Howard Gardner's multiple intelligences (1985). The CCP focuses on seeing mathematic instruction as closely tied to the human experience unlike the disconnected theories of the PFP. In this way the constructivist paradigm can be considered relatively synonymous with the cognitive cultural paradigm.

The reform in mathematics took on many forms but ultimately led to new programs, books and philosophies. The constructivist methodologies look different from program to program, philosophy-to-philosophy yet, there are many similarities. As in any constructivist classroom one will see (1) complex and relevant learning situations, (2) social dialogue, (3) multiple perspectives and multiple modes of learning, (4) student ownership in learning, (5) students who are self-aware of the knowledge they have constructed (Driscoll, 2005). A constructivist environment encourages students; to reflect on their learning, understand and be able to use their knowledge, think critically about learning and knowledge, and be able to reason through different problems. These are the conditions and goals one would see in a constructivist mathematics classroom.

With the research on cognition and teaching methodologies, the questions must be asked; Why do school districts resort back to the behaviorist theory of procedural formalist paradigm? Why do text book companies continue to publish programs that fall under these traditional methodologies?

In his book *Radical Constructivism in Mathematics Education*, (1992) Von Glasersfeld states, "Mathematics is not a discreet and separate enterprise unrelated to other varieties of knowledge and action. It is a social creation which changes with time and circumstances" (E.

von Glasersfeld, 1992, p.7). As we are in the wake of the change we find ourselves asking not only what the best methodologies are in which to teach mathematics, but also our role as the facilitator to educational discovery.

Complex and Relevant Learning Situations

The first part of the constructivist environment involves complex and relevant learning situations. This includes providing good problems for students to cover at a degree of depth that cannot be covered in a procedural formalist paradigm. This means that the educator must grab the student's attention with problems that are relevant in their personal world and will offer significant content. Investigations should include significant work with number sense including strategies for identifying relationships between the number operations, providing opportunities to graph, explain and reason with data, discovering and evaluating patterns, experiences with measurement and estimation, and development of spatial sense with two and three dimensional objects (Mokros, Russel, Economopoulos 1995). More importantly, many of these aspects should be included in one problem. In the PFP these ideas would be introduced individually and students would be given the "formula" and asked to practice this algorithm to mastery. In the constructivist classroom many ideas are needed to solve specific situations. The students are given a question that may require looking at various strands of mathematics. One such lesson in the fourth grade Bridges for Mathematics text asks the students to look at fans who went to a baseball game. They are asked to identify large numbers (in the millions) write about the numbers and have the ability to say the numbers. Then they are asked to compare various numbers in a table, order and make conclusions about the games. This lesson combines number skills with data analysis and it puts the children in a realistic situation where this skill would be meaningful.

This also speaks to the importance of knowing the difference between word problems and context problems (Fosnot & Dolk, 2002). An educator can create a problem that is intended to make students do the procedures the teacher is expecting. Problems in context open an array of possibilities to the children and are connected to the children's lives as well as the day's specific learning goal. Simply placing words on paper with the hopes that children will solve the problem does not make the problem contextual or relevant. Relevancy comes when the problem is given to the children as an investigation and the students are leading themselves and helping one another in the learning process.

Another example of looking at the complexity and depth of mathematics is the idea of introducing and teaching children multiplication through the array model. The array model of multiplication allows children to see the factors and the product visually, they can manipulate tiles and other tools through discovery and they can extend this knowledge into geometry instruction. This gives students the ability to connect a strong foundation in number with geometric properties and ideas.

Another idea for creating relevant and complex curriculum comes from the idea proposed by Arthur Hyde in *Comprehending Math, Adapting Reading Strategies to Teach Mathematics, K-6* (2006). For years reading instructors have been looking at strategies that help children become better readers. These strategies include questioning, making connections, visualization, inferring and predicting, determining importance and synthesizing. While this topic is vastly larger than what will be discussed here, it is easily connected to the constructivist classroom and would rarely be seen in the Procedural Formalist classroom.

In reading, asking questions involves children composing questions that will help them determine the various story elements in reading and leads to comprehension. Children are

encouraged to ask questions that can be both answered and that might spark other thinking and connections. Educators define these questions as either surface or “deep” which goes beyond the surface of the question to look at deeper reasons. A surface questions may be asked to determine a students understanding of specific details from the text such as setting, the character’s problem, etc. Deeper questions are intended to find additional questions and to deepen the child’s comprehension. These questions may not always be answerable. Questioning in mathematics is equally important. Questioning allows students the ability to look at mathematical concepts in a deeper context. Deep questions invite children to ask questions that may not always be answerable and provide the opportunity to look at a problem or situation more deeply. A surface question in mathematics would look at the specifics of a problem. “What are the factors of the number thirty-six?” Posing the question, “How are division and fractions related” would be a deep question and asks students to begin looking at the complexity and depth of our number system.

Making connections in reading has students connecting the text they are currently reading to other texts, the reader’s personal experiences or global issues. A child’s schema, or what they already know and understand, is brought to the surface and connected to new ideas. This can easily be incorporated in the mathematics classroom. Asking students to connect the various mathematical strands, connecting the situations their personal world or solving problems contextually that answer a real world question are similar. An example of a connection would be to ask students to look at the traditional multiplication algorithm and find the partial products in the array model. This encourages students to connect what they have learned about finding the product in an array method, apply this knowledge to the partial product method of multiplication and find the reasoning behind why the traditional algorithm works.

Visualization is an important skill in reading and in mathematics. In reading, the child is asked to visualize what they are reading. It is often referred to as a “mind” movie. This becomes an important skill in mathematics. The connection of visualization to geometry is easy to make. Students must be able to use visualization techniques when finding properties of two and three-dimensional objects. However, visualization is also a powerful tool in multiplication. After children have worked extensively with the array model for multiplication and division they are able to visualize the arrays when working independently and are more likely to keep their number sense when finding the product or quotient. This also applies to patterns and graphing.

Making inferences and forming predictions about the text is an essential strategy for understanding the modes of language. Students are often asked to “read between the lines” of the text to determine what the author means without explicitly writing it. A theme is an example of something that is inferred by the reader. Themes are rarely given to the reader. The reader must infer the theme of the story based on the details of the text. Students are also encouraged to make predictions about what might happen next in the text. “Mathematical situations are different from fiction and poetry, where language is supposed to evoke images and emotions” (Hyde, 2006, p.108). With this in mind consider the skill of estimation. This is a form of prediction and is an important skill. Using estimation and rounding can help children evaluate their mathematical thinking in and out of the classroom.

Finally, determining importance and synthesizing are easily found in reading and mathematics. Determining importance in reading requires the child to pull out the necessary information that will help them comprehend the text. They must distinguish between the “need to know” and “nice to know” information. They must determine which word, sentence and piece of the text is important to comprehend, form interest, analyze the text and ask questions.

Students must also be able to do this when working in mathematics. This is obvious when working with a contextual problem. Children must be able to determine the pieces of the problem that are necessary in order to solve the problem.

When synthesizing a child “combines new information with existing information to create something new” (Hyde, 2006, p.151). This is the process of placing all the parts together to solidify new information. As a child’s mathematical schema increases so does the child’s ability to apply this knowledge to new situations. As children navigate between the strands of mathematics they must use their prior knowledge in each of the areas to form new ideas. I return to the example of use of the array model of multiplication. If a child uses his schema about finding the product through an array, he can combine this prior knowledge with new ideas about and experiences with area to form effective ways in which to determine a shape’s area and to distinguish between the concept of area and perimeter. Synthesis happens when children begins to solve and explain with tables without prompting, are able to take concrete models and transfer them to abstract algorithms and evaluate patterns by engaging not only visuals but with numbers.

The connection between these reading strategies and mathematical thinking is powerful proof of the importance of constructivist thinking and the development of a relevant and complex curriculum.

Social Dialogue

Social dialogue is also a very relevant topic in the constructivist classroom. Because social interaction and negotiation of knowledge with another individual is an important piece of constructivism, one must consider the delivery of material that is used and the questions that are posed. The way in which a teacher poses an investigation, responds to an answer and teaches the children to communicate with one-another is extremely important in a constructivist

classroom.

Fosnot (2002) speaks of creating mathematical communities in the classroom, which is also an embedded piece of both the Investigations and Bridges in Mathematics programs. In a mathematical community both students and teachers are learners. The lines between teacher and student are blurred and the classroom as the children become the teacher and the teacher becomes the learner. This idea is embraced in the cognitive-cultural paradigm.

When creating a community of mathematical learners, the educator must simultaneously create a safe culture of learning and for the individual students while also encouraging a means social dialogue and mathematical thinking. In a classroom that values a student's learning the curriculum must not only focus on what is being taught but also how the student learns (Mokros, Russell, & Ecnomopoulos, 1995, p. 27). This can be determined by listening to the students discuss ways in which they solved a specific problem and focus on the children's mathematical reasoning.

A teacher can learn a great deal about a student's knowledge by simply listening or reading the explanation on how they constructed the answer. In the constructivist classroom the teacher's emphasis should be on understanding how the child determined the answers. It is here that an educator can truly hone in on the skills in which a student is lacking or excelling. Because "representing and solving the problem go hand in hand" (Mokros, Russell, & Ecnomopoulos, 1995, p. 70) it becomes increasingly important to emphasize assessing a student's thinking as well the answer.

When using Bridges and Investigations curriculum students are encouraged to use the "think, pair, share" strategy. When an investigation is posed students use quiet time to independently think about the problem. They use this time to determine the important pieces of

the problem, the strategies they may use to solve a problem, and the direction they will take. Ample think time must be given. The students then share their thinking. The sharing can be done with a partner, in a small group or with the whole class. The environment must be safe for the child to fully share the thinking. It would be the responsibility of the teacher to create a safe way to share and a safe and respectful way to disagree with one's thinking. Sharing confused thinking or the wrong answer is inevitable. Teaching children how to disagree with a wrong answer and, reversely, how to confront their own misguided thinking are not only important skills in the mathematics classroom but in everyday life.

When posing questions to students a teacher can either encourage or destroy a child's natural desire to investigate a problem. Consider the following questions that were asked in *Beyond Arithmetic* (Mokros, Russell, & Economopoulos, 1995, p. 19):

1. 42×37
2. Nora has 42 people coming to her party. She wants to give each of them a bag of 37 peanuts. How many peanuts does she need to buy?
3. Each year the fifth grade takes a trip to Washington, D.C. They have a car wash to raise money to pay for their expenses. This year there are 42 students in fifth grade. The bus fare and lunch will cost \$37 per student. How much money do they need to raise to pay for the expenses of all the students?

On the surface, one may question the ability to deeply consider question one. This would be a question one might expect to see in a Procedural-Formalist classroom and could be answered by the application of the traditional algorithm. However, if one were to look at this problem in a constructivist classroom students would be expected to share their developed strategies and analyze the results. Manipulatives, graph paper, a cooperative environment and

the expectation that the students will be constructing methods for analysis transform this into a very appropriate question that is based on the desire for in-depth understanding for the student. In comparison, question 3 may look easily identifiable as a contextual problem. However, if you learned that this was one of many that was intended to be practice for applying the traditional algorithm one must reconsider the validity of the question. One must also consider how contextually relevant this question is to the child that has never traveled. The quality of the questions depends on the teacher's instructional plans and goals, and the delivery of the problem, which ties back to the social dialogue and learning environment used in the classroom.

In *Young Mathematicians at Work* (2002), Fosnot and Dolk discuss the idea of Math Congress. This is an open forum for discussion of the day's work. It is an important time for students to share work, discuss ideas, prove hypothesis, and discuss strategies with one another. It is during this time that learners "defend their thinking." "Once again we as teachers are on the edge. We must walk the line between structure and the development of mathematics, and between the individual and the community. As we facilitate discussions, as we decide which ideas to focus on, we develop the community's norms and mores with regards to mathematics, and we stretch and support individual learners. We move the community toward the horizon, and we enable individuals to travel their own path." (Fosnot, & Dolk, 2002, p.34)

Open discourse between students allows students to clarify their knowledge. "Rethinking, rewriting, and refining are as important to the process of solving problems and understanding mathematics as they are to the writing process." (Mokros, Russell, & Ecnomopoulos, 1995, p.49) In the constructivist classroom the teacher becomes the audience as the students explain to the teacher, each other and the classroom as a whole.

Multiple Perspectives and Modes of Learning

Once the teacher has handed learning over to the students, multiple perspectives on learning will occur. When the teacher realizes that there is no single approved way to solve advanced multiplication students will create and construct strategies that are meaningful to them. When students are given opportunities to investigate measurement schema will develop and open the way for new knowledge. Including and inviting multiple perspectives on learning is at the heart of the constructivist classroom. It is also important to consider different students' modes of learning.

Many theorists have composed theories on the multiple modes of learning. One of these theories is Gardner's Multiple Intelligences. (Gardner 1983) Gardner proposed that individuals' minds are composed differently. Gardner's work is closely related to ideas in a constructivist classroom. Gardner deduced that teachers could no longer assume that children all learn the same way. He proposed that student's minds gravitate toward specific domains. Gardner argued that student could be more successful if lessons were presented and assessed in various modes. These modes can be specifically implemented into the mathematics classroom and include Visual – Spatial, Bodily-Kinesthetic, Musical, Interpersonal, Intrapersonal, Linguistic and, of course, Logical-Mathematical. The connection to almost every mode is an easy tie-in to the constructivist environment.

Student Ownership and Self-Awareness of Learning.

Because students are constructing and sharing their learning in a constructivist classroom, student ownership of their personal learning and self-awareness are extremely important. As students begin to construct knowledge and use this knowledge to solve difficult problems they begin owning this knowledge. They also begin to become meta-cognitive in their learning which makes them able to monitor their thinking as they learn. Students begin to be able to recognize

the conditions and limitations of the problems posed. They are able to determine if the amount of information given for the problem is sufficient, the strategies to come to “the answer,” and determine whether there is a single answer to the problem. They are able to value themselves as a learner, and, in turn, offer their knowledge as a teacher.

From Theory to Practice

In a book by Graeber, Valli and Newton, (2011) case studies are closely recorded and preserved to teach pre-service and to enhance experienced teachers methodologies. In the preface of the book the author lists the qualities looked for in the teaching of different mathematical lessons. Of these qualities listed as good practices are drawing on a student’s schema, using multiple representations, and discussing one’s reasoning behind solving a problem. These are all strategies one can expect to see in a room built on a constructivist philosophy and the lessons were all tied to Common Core State Standards.

Case one involved giving 5th grade students a problem involving eight pounds of apples to use to in order to make pies. The question asked was “Rochelle bought eight pounds of apples for pies. If each apple weighs four ounces, how many apples did she buy?” (Graeber, Balli, & Newton, 2011, p 26). Students then discussed the best way to “tackle” the problem. One student suggested drawing four-ounce apples until eight pounds was reached. The student ran out of room on the board to complete drawing the apples. When the teacher noticed that the student was using repeated addition to complete the problem she asked the student if there was a “shortcut” that could be used to find the answer. Another student suggested that they could do eight times four. At this point the teacher interjected with “Because I am going to do this eight times. Eight times four is...?”

Whether this lesson presents itself as purely constructivist remains unclear. However, the

teacher allows each child the flexibility to create a strategy, the opportunity to discuss how they solved the problem and a forum for new ideas from the other students.

Lessons in the Bridges curriculum are very similar. Problems are posed to the students in various contexts. The problems introduced in a way to help students construct knowledge about the different strands of mathematics. Often, more than one strand must be considered to solve the problem. Students are then encouraged to choose a manipulative to help himself or herself, work with a friend then share out.

Seeing how these lessons invigorated the students made it clear to the teachers who were on board with this philosophy to continue down the constructivist path. As with any major change however, there were skeptical teachers and several unconvinced parents. Fortunately we had been working with a consultant who prepared us for some questions and we were encouraged to read *Beyond Arithmetic* by Mokros, Russell and Economopoulos (1995). Many of our answers came from this text. The consultant we worked with was trained specifically in the Bridges in Mathematics program and she stated that her philosophy was constructivist in nature. Her training included strategies for teaching number sense in a constructivist manner and helping parents understand the importance of a constructivist environment.

The first question that needed answering was the question on learning basic mathematics facts. Teachers were asked why our strategy of teaching basic mathematics facts was not with fact cards and memorization. This question was answered by asking determining the ultimate goal of knowing these facts. The conclusion was that memorization is not the goal. Fluency is the goal. Students in our constructivist classrooms were learning new strategies to determine basic facts and receiving ample experiences to practice and form strategies. “The important thing is that students be able to use their knowledge of numbers to fluently construct calculations

that may be difficult to remember. If you don't remember 9×8 , you can easily derive it by using your knowledge of 10×8 , then subtracting the extra 8. A fluent mathematics user will apply strategies like this often, and in the process will be learning a great deal about mathematical relationships" (Mokros, Russell, & Economopoulos, 1995, p.72). Also, teachers and families must remember that just a small portion of mathematics includes fact memorization. What's more important is that students are building knowledge of the number system and the ability to look at all the different similarities, variances and manners in which numbers can be manipulated. This will build fluency in facts.

The next inquiry came from the allowance of student created strategies and the elimination of teaching of the traditional algorithm as the introduction to a concept. Because borrowing, carrying and other procedures were taught to many parents and teachers when they were learning mathematics, we were told that these methods should be "enough" by reluctant believers. The reason this is a dangerous process is due to two factors, maintenance of a child's number sense and maintaining mathematical thinking. Learning and repeating the steps in traditional algorithm is the very essence of the procedural-formalist protocol. However, these steps are not always the best and most effective for our students.

"Borrowing, carrying, the procedure for long division - they're not universal. In other countries and at other times in our own country, students have been taught different and equally effective algorithms for the basic operations. Constructing effective algorithms, ones that can be used efficiently in a range of different situations, is in itself an important element of mathematical thinking. Students who invent algorithms that are easy to use are doing significant mathematical work. On the other hand, applying some else's algorithms to solve a problem –

especially if you have no understanding of how or why these algorithms work – is not ‘doing mathematics.’” (Mokros, Russell, Econompoulous, 1995, p.73)

Teaching students “yours is not to wonder why, just invert and multiply,” and other such “tricks” in mathematics. When we keep in mind the other benefits of the constructivist classroom (communication, student empowerment) it adds credence to the importance of students constructing meaning of and creation of strategies to complete the problems PFP could only solve with the traditional equations. The constructivist classroom tries to create mathematical thinkers, not create process memorizers.

The use of student-constructed strategies also helps a child maintain his number sense. When a child is learning to simply follow the steps to complete a problem he will not see the larger idea in the problem. For example, if a child is learning to complete a 2 digit by 2 digit multiplication problem with the traditional algorithm they are asked to carry tens to the tens place then add it to the numbers multiplied they are often confused as to why. This is such an abstract concept. Children who only learn the algorithms are frequently unable to answer the why and how questions about the process. This destroys the ability to “easily” estimate and mentally solve problems. It is also important to give children the time and feedback needed to understand what they are looking for and to find meaning. Sharing of strategies and thoughts on processes used is very valuable. It allows the students to see the different ways in which problems can be solved. In the Investigations program they speak specifically about not teaching the traditional algorithm. They believe that teaching children the algorithm is detrimental to children’s growth in number sense and “fluency with the number system.”

Finally, teachers wondered if they might be able to combine some of the new methodologies with the traditional teaching procedures to get the ‘best of both worlds’? This

teaching which combines the Procedural Formalist Paradigm with the Cognitive-Cultural Paradigm will confuse students who are trying to make sense of mathematics. Mokros, Russel and Economopoulos (1995) equate this teaching style with asking children to create their own strategies then informing them that their own strategy is not the correct strategy to use. In 1993 Constance Kamii and Barbara Lewis provided evidence that the teaching of the standard algorithm untaught what the student knew about place value and was a deterrent in learning and developing number sense. When children are learning to add numbers in the traditional manner they are taught to add the numbers in isolation. 426 plus 162 invites the child to add 2 and 6, then 6 and 2 and finally 4 and 1. They are not thinking of adding the ones, tens then hundreds when doing the algorithm. The constructivist child will add ones, tens then hundreds. If a teacher allows a child to invent a strategy to add these numbers then shows them the traditional algorithm, all knowledge gained about the number system and that child's specific number sense is undone with the assumption that the teacher's method must be the 'correct' method. This is dangerous not only to the child's growing knowledge about mathematics and number sense but also to a child's confidence in their own learning.

Through the above-described methodologies, an educator in the area of mathematics can make a difference in a child ability to think mathematically and have confidence in building mathematically proficient students.

Chapter 3

Research related to this topic asks the question how does one define success in the area of creating mathematicians. Does creating a mathematician mean creating students who can simply solve a set of algorithms or does it mean creating students who can take strategies created in order to solve complex problems and apply this knowledge independently.

In a constructivist environment the child's thinking about mathematics is just as, if not more important than the right answer on a question. As a teacher begins to build confidence in working in this environment, they notice how using the constructivist methodologies increase a child's effectiveness in mathematics and confidence in learning. I noticed this in my own classroom. Children who were unsuccessful in mathematics prior to the use of the Investigations and Bridges in Mathematics began feeling confident in the work they were doing. Students who did not understand the algorithms began creating their own strategies for the basic operations and were confident in sharing these strategies with others. Once the sharing began other students used that knowledge and began creating their own ways to solve problems.

Because this was an entirely new way of thinking, teachers needed to become the learners themselves. This came naturally for some, but proved more difficult to others. Our school noted the need to help our teachers embrace this philosophy and to become more efficient teachers. School A teachers also began work with a consultant on how to teach in a constructivist manner. We worked as a staff and other teachers around our district to develop knowledge about the new curriculum and effective teaching practices. We not only learned about the constructivist philosophy but how to engage our learners with effective communication. We learned that when disequilibrium was happening with our students this is when they began truly learning a concept. Teachers were no longer expected to teach every

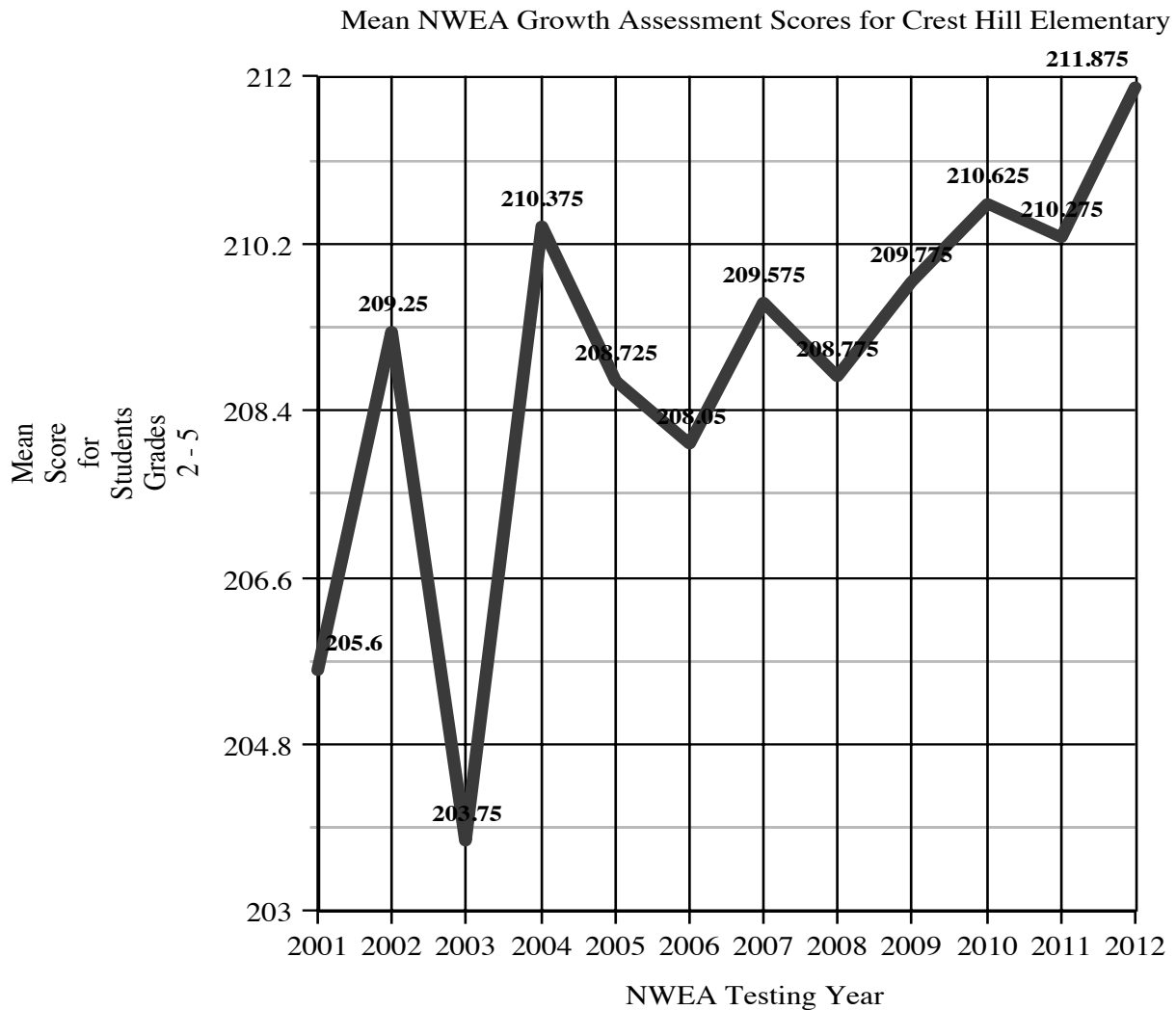
intricate algorithm and skill in mathematics but were expected to expose children to a multitude of problems that would allow a student to deeply think about our number system and develop connections and sound strategies that would prove useful in many situations.

As we began making this change, there was a notable change in educators' opinions about students and teachers as mathematicians. Many teachers were excited about strategies students were using with the basic operations. Students were using open number lines to solve subtraction problems, investigating with manipulatives, building arrays and using partial products to find products, and building special sense and developing connections between the array model for multiplication and geometric shapes after creating and exploring the properties of area and perimeter.

We also noticed that our students were becoming more confident in their problem solving strategies. Many of my students began sharing more with the whole class and were less phased when their mathematical thinking was questioned or when a given answer was incorrect. They seemed to begin viewing these challenges as a part of the learning system.

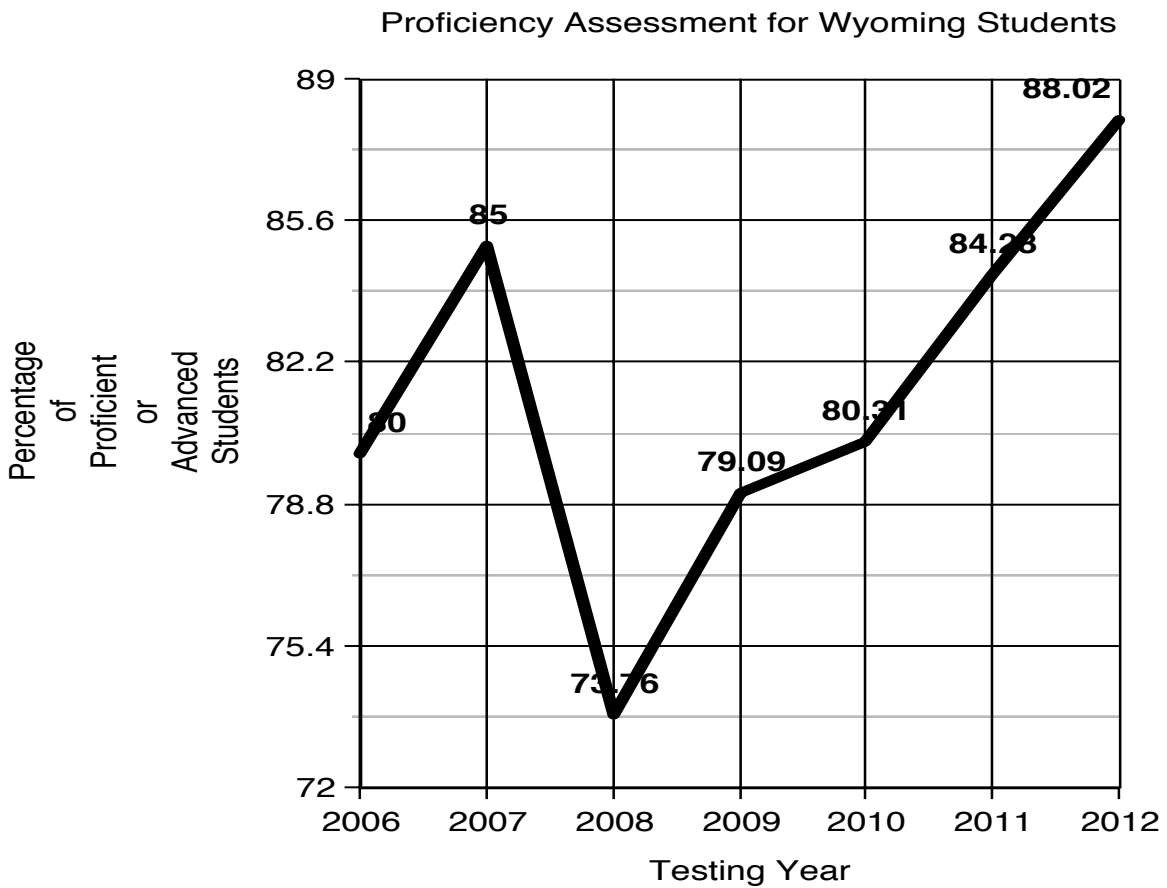
Parents seemed to have had the most difficulty adapting to the new changes. Over and over at parent teacher conferences I had discussions with parents about why their children were not learning the same way in which they did. Fortunately, we had been working with consultants who prepared us for just these questions and had gained valuable knowledge about our teaching style. The most frequently asked questions were about learning basic facts and learning the traditional algorithm. Because we were teaching our students different strategies to become fluent with their basic operation facts we had questions as to why we were not just using flashcards or other memorization techniques. We explained the difference between conceptual knowledge, fluency and memorization.

After we began analyzing our NWEA data we saw some significant growth in our students. In 2001, our mean score on this assessment was 205.6. Between the years of 2002 and 2005 we saw some very inconsistent numbers. By the end of our first year of using the Investigations program in 2006 we saw our average grow to 208.05. After a few years of developing our knowledge of the constructivist methods we were using we grew to an average of 211.875. Even knowing that the NWEA assessment is a very traditionally driven assessment in the questions offered, we felt proud that our students were able to apply their own strategies and knowledge of mathematics to this assessment.



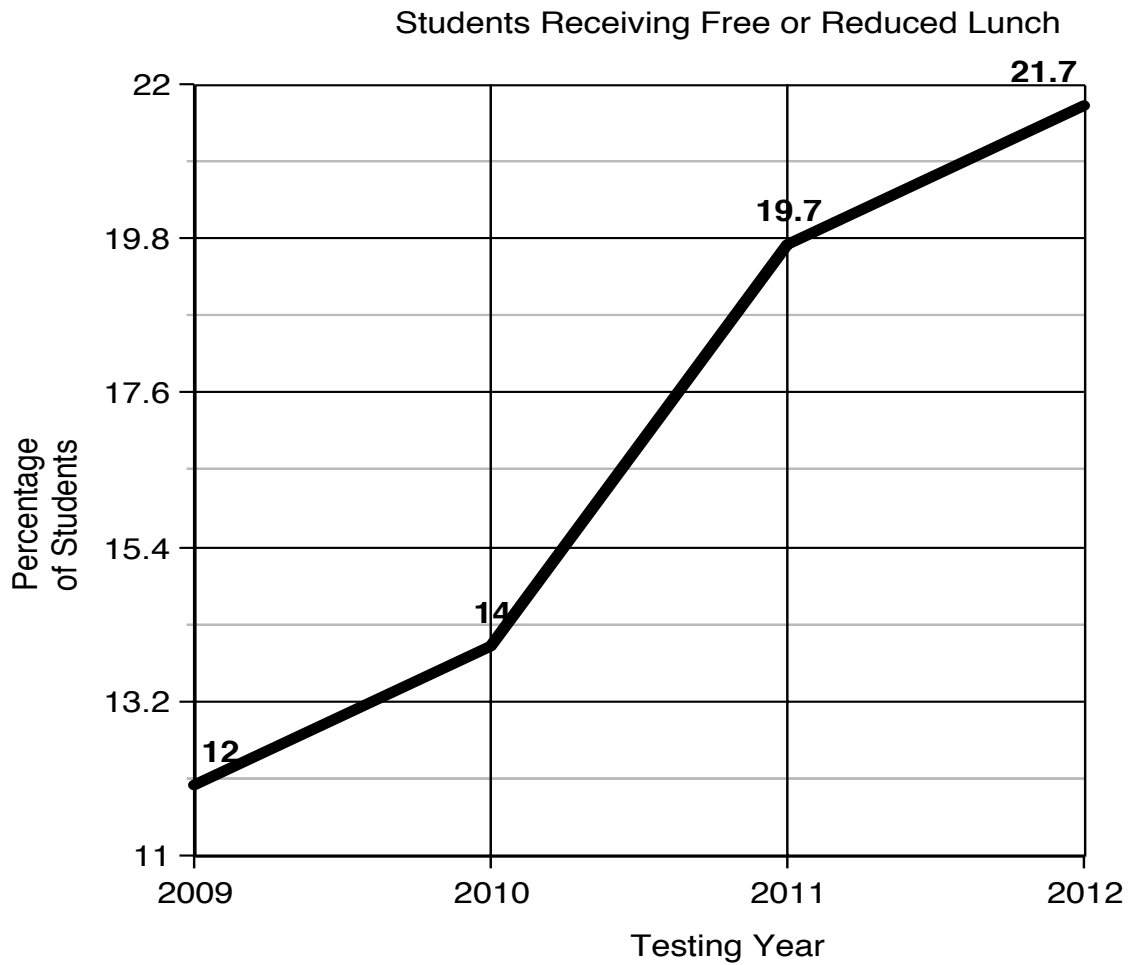
Another assessment we take in Wyoming is the PAWS or the Proficiency Assessment for

Wyoming Students. This assessment was created to measure out Adequate Yearly Progress due to the No Child Left Behind law. The PAWS assessment is both constructed response and multiple choice. It gives children a score based on a rubric for the constructed response items and gives the child credit for their mathematical thinking. The first year we took this test was the first year we had fully implemented the Investigations program. 80 percent of our students were proficient or advanced on this assessment given to third, fourth and fifth grade students. The following year we were at 85 percent proficient or advanced. In 2009 we had changed our program to Bridges in Mathematics and we noticed a significant drop in our children’s achievement. However, each year we grew more students and by the end of the 2012 school year 88 percent of our students were proficient or advanced.



This alone was exciting to see. When we began looking at our number of students who

were receiving free or reduced lunch we felt even more confident that we chose the right methods to teach our students. Generally, the greater the population of free and reduced lunch students will lower the scores on assessments. This is due to many factors. Students receiving free and reduced lunches may often have less support at home and value their education less than students of affluent families. Looking at our data, we noticed that although our population was changing, our test scores continued to rise.



Based on the evidence we have seen at School A, constructivist methods seem to be growing our students more consistently.

During the writing of this paper, our school board met and voted to adopt the new mathematics program. Teachers will no longer be allowed to teach with a constructivist curriculum. Teachers will be given a student work book with over 400 pages of worksheets to learn mathematics. As a constructivist teacher, I am truly disheartened by the decision. I wrote to every school board member encouraging him or her to deny adoption this program. Few responded to my request. Those who did informed me that the representative from the My Math program did a very effective job of convincing the educators on the adoption committee that My Math will teach the Common Core State Standards (CCSS). One of my arguments against My Math was that companies are currently in the process of aligning their programs to the CCSS. If we had given these companies just a bit more time Bridges in Mathematics will be publishing a finished edition which meets the Common Core.

Also, during the writing of this paper, news specifically about the CCSS has been widespread. National Public Radio has run several stories questioning and explaining the CCSS. Many educators are beginning to truly question the validity of these standards. There are growing questions as to the involvement of our government in state education and many question the appropriateness of these standards. Some states which had adopted these standards have now rejected them and have gone back to their state standards.

It seems to me that each time educators and legislators panic about test scores they revert back to what they knew as a learner. Look at the history in Chapter 1. Methodologies change from one paradigm to the other, often leaving children behind in the process. We are confusing our children. One year they are learning the traditional algorithms, the following year they are thrown into a constructivist new program. We need to consider the children as learners, not just as test takers.

This in mind, I feel that our education system also needs to consider a way to assess students' knowledge in a meaningful and measurable manner. Wyoming changed the PAWS assessment in the 2013 school year to make it easier to measure from the test company's perspective. All constructed response questions were eliminated. Students are no longer given credit for their mathematical thinking, only for finding the correct answer. This is a step in the wrong direction.

As I look forward to my children's education and my profession as an educator I have begun to develop a deep concern. The Common Core State Standards are not the enemy of the teacher and student. The programs that companies develop seem to be the issue. One of the goals of the Common Core is to help teachers with major shifts in how education happens in the United States. One of these shifts is to promote conceptual understanding prior to asking students to perform procedural skill work. This solid conceptual understanding seems to be being skipped in the math program our district has chosen.

It is the responsibility of each teacher to be careful to assess a student's conceptual knowledge prior to encouraging procedural efficiency. While procedural efficiency and fluency in number operations are important, they need not be the entire focus of a math curriculum. As a teacher who personally struggled with math, I am proof of the dangers of using the procedural-formalist methodology as the core instruction program. I use this knowledge as I teach math and am constantly cognizant of just how an introduction to any topic can make or break a student's mathematical education. It is my job as a parent and a teacher to be aware of my own children and my student's level of preparedness before they are even introduced to an algorithm.

In this day and age, children are expected to be skilled in many areas. The Procedural-Formalist theory of the "sage on the stage" can no longer be employed in our classrooms. We

want children to be critical thinkers and to have the skill set to solve world issues they have not yet been exposed to. We want children to develop educational independence and to become successful. We want children to become socially aware of themselves and other individuals. We have many goals for children. Educators can no longer ignore the important set of skills that come from the constructivist classroom. We are no longer afforded the laziness that accompanies the traditional Procedural-Formalist paradigm. It takes independence, ingenuity, and confidence to construct one's own knowledge. Constructivism involves taking learning risks and the humility that comes from discovering that one's answer is incorrect. Not only is this powerful in the world of mathematics, this is powerful in the development of socially competent people.

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